MAHARASHTRA STATE BOARD OF TECHNICAL EDUCATION

(Autonomous) (ISO/IEC - 27001 - 2005 Certified)

(150/1EC - 27001 - 2005 Certifica)

WINTER – 2018 EXAMINATION

Subject Name: Applied Mathematics Model Answer

Subject Code:

22224

Important Instructions to Examiners:

- 1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
- 2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
- 3) The language errors such as grammatical, spelling errors should not be given more importance (Not applicable for subject English and Communication Skills).
- 4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
- 5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answer and model answer.
- 6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
- 7) For programming language papers, credit may be given to any other program based on equivalent concept.

Q. No.	Sub Q. N.	Answer	Marking Scheme
1.		Solve any <u>FIVE</u> of following:	10
	a)	If $f(x) = x^3 - x$, find $f(1) + f(2)$	02
	Ans	$f(x) = x^3 - x$ $\therefore f(1) = (1)^3 - (1) = 0$	1/2
		$\therefore f(2) = (2)^3 - (2) = 6$	1/2
		$\therefore f(1) + f(2) = 0 + 6$	1/2
		$\therefore f(1) + f(2) = 6$	1/2
	b) Ans	State whether the function $f(x) = x^3 - 3x + \sin x + x \cdot \cos x$ is even or odd. $f(x) = x^3 - 3x + \sin x + x \cdot \cos x$ $\therefore f(-x) = (-x)^3 - 3(-x) + \sin(-x) + (-x) \cdot \cos(-x)$ $= -x^3 + 3x - \sin x - x \cdot \cos x$	02
		$= -(x^3 - 3x + \sin x + x \cdot \cos x)$ $= -(x^3 - 3x + \sin x + x \cdot \cos x)$	1/2
		=-f(x)	1/2
		∴ Function is odd.	1/2
	c)	Find $\frac{dy}{dx}$ if $y = e^{2x} \cdot \log(x+1)$	02



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Q. No.	Sub Q. N.	Answer	Marking Scheme
1.	c) Ans	$y = e^{2x} \cdot \log(x+1)$ $\therefore \frac{dy}{dx} = e^{2x} \cdot \frac{1}{x+1} + \log(x+1) \cdot e^{2x} \cdot 2$ $= \frac{e^{2x}}{x+1} + 2e^{2x} \log(x+1)$	1+1
	Ans	Evaluate $\int \left(e^{2x} + \frac{1}{1+x^2}\right) dx$ $\int \left(e^{2x} + \frac{1}{1+x^2}\right) dx$	02
		$=\frac{e^{2x}}{2}+\tan^{-1}x+c$	1+1
		Evaluate $\int \frac{dx}{9x^2 - 16}$ $\int \frac{dx}{9x^2 - 16} = \frac{1}{9} \int \frac{dx}{x^2 - \frac{16}{9}}$	02
		$=\frac{1}{9}\int \frac{dx}{x^2 - \left(\frac{4}{3}\right)^2}$	1/2
		$= \frac{1}{9} \frac{1}{2 \cdot \frac{4}{3}} \log \left(\frac{x - \frac{4}{3}}{x + \frac{4}{3}} \right) + c$ $= \frac{1}{24} \log \left(\frac{3x - 4}{3x + 4} \right) + c$	1 1/2
		OR	
		$\int \frac{dx}{9x^2 - 16} = \int \frac{dx}{(3x)^2 - (4)^2}$ $= \frac{1}{2(4)} \cdot \frac{1}{3} \log \left(\frac{3x - 4}{3x + 4} \right) + c$	1
		$= \frac{1}{24} \log \left(\frac{3x-4}{3x+4} \right) + c$	1



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_		ame. Applied Mathematics Model Answer Subject Code. 22	
Q. No.	Sub Q. N.	Answer	Marking Scheme
1.	f)	Find the area enclosed by the curve $y = x^3$, x-axis and the ordinates $x = 1$ and $x = 3$	02
	Ans	Area $A = \int_{a}^{b} y dx$	
		$= \int_{1}^{3} x^{3} dx$ $= \left\lceil \frac{x^{4}}{4} \right\rceil_{1}^{3}$	1/2
		$= \frac{(3)^4}{4} - \frac{(1)^4}{4}$	1/2
		$= \frac{81}{4} - \frac{1}{4}$ $= 20$	1/2
	g)	Show that the root of $x^3 - 9x + 1 = 0$ lies between 2 and 3.	02
	Ans	$f(x) = x^3 - 9x + 1$	
		$f(2) = (2)^3 - 9(2) + 1 = -9 < 0$	1
		$f(3) = (3)^3 - 9(3) + 1 = 1 > 0$	1
		∴ Root lies between 2 and 3	
2.		Solve any <u>THREE</u> of the following:	12
	a)	If $x^2 + y^2 + 2xy - y = 0$ find $\frac{dy}{dx}$ at (1,2)	04
		$x^2 + y^2 + 2xy - y = 0$	
		$2x + 2y\frac{dy}{dx} + 2\left(x\frac{dy}{dx} + y\right) - \frac{dy}{dx} = 0$	2
		$2x + 2y\frac{dy}{dx} + 2x\frac{dy}{dx} + 2y - \frac{dy}{dx} = 0$	
		$2y\frac{dy}{dx} + 2x\frac{dy}{dx} - \frac{dy}{dx} = -2x - 2y$	
		$\left(2y+2x-1\right)\frac{dy}{dx} = -2x-2y$	1
		$\frac{dy}{dx} = \frac{-2x - 2y}{2y + 2x - 1} = \frac{-2(x + y)}{2y + 2x - 1}$	1
		$\left(\frac{dy}{dx}\right)_{(1,2)} = \frac{-2(1+2)}{2(2)+2(1)-1} = \frac{-6}{5} \qquad \text{OR} -1.2$	1



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	,	ame: Applied Mathematics <u>Model Answer</u> Subject Code: 222	
Q. No.	Sub Q. N.	Answer	Marking Scheme
2.	b)	If $x = a(2\theta - \sin 2\theta)$, $y = a(1 - \cos 2\theta)$ find $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$	04
	Ans	u_{λ}	
		$x = a(2\theta - \sin 2\theta) \qquad \qquad y = a(1 - \cos 2\theta)$	
		$\therefore \frac{dx}{d\theta} = a(2 - 2\cos 2\theta)$ $\frac{dy}{d\theta} = 2a\sin 2\theta$	1+1
		$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$ $\therefore \frac{dy}{dx} = \frac{2a\sin 2\theta}{2a(1-\cos 2\theta)} = \frac{\sin 2\theta}{(1-\cos 2\theta)} \text{OR} \frac{dy}{dx} = \frac{\sin 2\theta}{(1-\cos 2\theta)} = \frac{2\sin \theta \cos \theta}{2\sin^2 \theta} = \cot \theta$	
		$\therefore \frac{dy}{dx} = \frac{2a\sin 2\theta}{2a(1-\cos 2\theta)} = \frac{\sin 2\theta}{(1-\cos 2\theta)} \text{OR} \frac{dy}{dx} = \frac{\sin 2\theta}{(1-\cos 2\theta)} = \frac{2\sin \theta\cos \theta}{2\sin^2 \theta} = \cot \theta$	1
		at $\theta = \frac{\pi}{4}$	
		$\therefore \frac{dy}{dx} = \frac{\sin 2\left(\frac{\pi}{4}\right)}{\left(1 - \cos 2\left(\frac{\pi}{4}\right)\right)} = \frac{\sin\left(\frac{\pi}{2}\right)}{\left(1 - \cos\left(\frac{\pi}{2}\right)\right)}$	
		$\therefore \frac{dy}{dx} = \frac{1}{1 - 0} = 1$ OR $\frac{dy}{dx} = \cot \frac{\pi}{4} = 1$	1
	c)	Find the maximum and minimum value of $y = x^3 - \frac{15}{2}x^2 + 18x$	04
	Ans	Let $y = x^3 - \frac{15}{2}x^2 + 18x$	
		$\therefore \frac{dy}{dx} = 3x^2 - 15x + 18$	1/2
		$\therefore \frac{d^2y}{dx^2} = 6x - 15$	1/2
		Consider $\frac{dy}{dx} = 0$	
		$3x^2 - 15x + 18 = 0$	1/2
		$x^2 - 5x + 6 = 0$	1/
		$\therefore x = 2 \text{ or } x = 3$ at $x = 2$	1/2
		$\frac{d^2y}{dx^2} = 6(2) - 15 = -3 < 0$	1/2
		\therefore y is maximum at $x = 2$	
		$y_{\text{max}} = (2)^3 - \frac{15}{2}(2)^2 + 18(2) = 14$	1/2



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Q. No.	Sub Q. N.	Answer	Marking Scheme
			Scheme
2.	c)	at $x=3$	1/2
		$\frac{d^2y}{dx^2} = 6(3) - 15 = 3 < 0$	72
		\therefore y is minimum at $x = 3$	
		$y_{\min} = (3)^3 - \frac{15}{2}(3)^2 + 18(3) = 13.5$	1/2
			, 2
	d)	A beam is bent in the form of the curve $y = 2 \sin x - \sin 2x$.	04
		Find the radius of curvature of the beam at the point $x = \frac{\pi}{2}$	
	Ans	$y = 2\sin x - \sin 2x$	
		$\therefore \frac{dy}{dx} = 2\cos x - 2\cos 2x$	1/2
		$\therefore \frac{d^2y}{dx^2} = -2\sin x + 4\sin 2x$	1/2
		ux	1/2
		$\left(\frac{dy}{dx}\right)_{\left(x=\frac{\pi}{2}\right)} = 2\cos\left(\frac{\pi}{2}\right) - 2\cos\left(\frac{\pi}{2}\right) = 2(0) - 2(-1) = 2$	1/
		$\left(\frac{d^2y}{dx^2}\right)_{\left(x=\frac{\pi}{2}\right)} = -2\sin\left(\frac{\pi}{2}\right) + 4\sin 2\left(\frac{\pi}{2}\right) = -2(1) + 4(0) = -2$	1/2
		$\therefore \text{ Radius of curvature is } \rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$	
		$\left[1+(2)^{2}\right]^{\frac{3}{2}}$	1
		$\therefore \rho = \frac{\left[1 + (2)^2\right]^{\frac{3}{2}}}{-2}$ $\therefore \rho = -5.59$ $\therefore \rho = 5.59$	1
		$\therefore \rho = -5.59$	1
		$\therefore \rho = 5.59$	
3.		Solve any <u>THREE</u> of the following:	12
	a)	Find the equation of tangent and normal to the curve $2x^2 - xy + 3y^2 = 18$	04
		at the point $(3,1)$	
	Ans	$2x^2 - xy + 3y^2 = 18$	
		$4x - \left(x\frac{dy}{dx} + y\right) + 6y\frac{dy}{dx} = 0$	1/2
		Dago No.	



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Q. No. Sub No. Q. N. Answer Mark Sche 3. a) $4x - x \frac{dy}{dx} - y + 6y \frac{dy}{dx} = 0$ $-x \frac{dy}{dx} + 6y \frac{dy}{dx} = -4x + y$ $(-x + 6y) \frac{dy}{dx} = -4x + y$ $\frac{dy}{dx} = \frac{-4x + y}{-x + 6y}$ at (3,1) Slope of tangent $= \frac{dy}{dx} = \frac{-4(3) + 1}{-3 + 6(1)} = \frac{-11}{3}$ Slope of normal $= \frac{-1}{\frac{dy}{dx}} = \frac{3}{11}$ Equation of tangent $y - y_1 = m(x - x_1)$ $y - 1 = \frac{-11}{3}(x - 3)$ $3y - 3 = -11x + 33$ $11x + 3y - 36 = 0$	
$4x - x\frac{d}{dx} - y + 6y\frac{d}{dx} = 0$ $-x\frac{dy}{dx} + 6y\frac{dy}{dx} = -4x + y$ $(-x + 6y)\frac{dy}{dx} = -4x + y$ $\frac{dy}{dx} = \frac{-4x + y}{-x + 6y}$ at (3,1) Slope of tangent = $\frac{dy}{dx} = \frac{-4(3) + 1}{-3 + 6(1)} = \frac{-11}{3}$ Slope of normal= $\frac{-1}{\frac{dy}{dx}} = \frac{3}{11}$ Equation of tangent $y - y_1 = m(x - x_1)$ $y - 1 = \frac{-11}{3}(x - 3)$ $3y - 3 = -11x + 33$ $11x + 3y - 36 = 0$	_
$-x\frac{dy}{dx} + 6y\frac{dy}{dx} = -4x + y$ $(-x + 6y)\frac{dy}{dx} = -4x + y$ $\frac{dy}{dx} = \frac{-4x + y}{-x + 6y}$ at (3,1) Slope of tangent = $\frac{dy}{dx} = \frac{-4(3) + 1}{-3 + 6(1)} = \frac{-11}{3}$ Slope of normal= $\frac{-1}{\frac{dy}{dx}} = \frac{3}{11}$ Equation of tangent $y - y_1 = m(x - x_1)$ $y - 1 = \frac{-11}{3}(x - 3)$ $3y - 3 = -11x + 33$ $11x + 3y - 36 = 0$	
$(-x+6y)\frac{dy}{dx} = -4x + y$ $\frac{dy}{dx} = \frac{-4x + y}{-x+6y}$ at (3,1) Slope of tangent = $\frac{dy}{dx} = \frac{-4(3)+1}{-3+6(1)} = \frac{-11}{3}$ Slope of normal= $\frac{-1}{\frac{dy}{dx}} = \frac{3}{11}$ Equation of tangent $y - y_1 = m(x - x_1)$ $y - 1 = \frac{-11}{3}(x - 3)$ $3y - 3 = -11x + 33$ $11x + 3y - 36 = 0$	
$\frac{dy}{dx} = \frac{-4x + y}{-x + 6y}$ at (3,1) Slope of tangent = $\frac{dy}{dx} = \frac{-4(3) + 1}{-3 + 6(1)} = \frac{-11}{3}$ V2 Slope of normal = $\frac{-1}{\frac{dy}{dx}} = \frac{3}{11}$ Equation of tangent $y - y_1 = m(x - x_1)$ $y - 1 = \frac{-11}{3}(x - 3)$ $3y - 3 = -11x + 33$ $11x + 3y - 36 = 0$	
at (3,1) Slope of tangent = $\frac{dy}{dx} = \frac{-4(3)+1}{-3+6(1)} = \frac{-11}{3}$ Slope of normal= $\frac{-1}{\frac{dy}{dx}} = \frac{3}{11}$ Equation of tangent $y - y_1 = m(x - x_1)$ $y - 1 = \frac{-11}{3}(x - 3)$ $3y - 3 = -11x + 33$ $11x + 3y - 36 = 0$	⁄2
Slope of normal= $\frac{-1}{\frac{dy}{dx}} = \frac{3}{11}$ Equation of tangent $y - y_1 = m(x - x_1)$ $y - 1 = \frac{-11}{3}(x - 3)$ $3y - 3 = -11x + 33$ $11x + 3y - 36 = 0$	
Slope of normal= $\frac{-1}{\frac{dy}{dx}} = \frac{3}{11}$ Equation of tangent $y - y_1 = m(x - x_1)$ $y - 1 = \frac{-11}{3}(x - 3)$ $3y - 3 = -11x + 33$ $11x + 3y - 36 = 0$	⁄2
Equation of tangent $y - y_1 = m(x - x_1)$ $y - 1 = \frac{-11}{3}(x - 3)$ $3y - 3 = -11x + 33$ $11x + 3y - 36 = 0$	⁄2
$y-1 = \frac{-11}{3}(x-3)$ $3y-3 = -11x+33$ $11x+3y-36=0$	
3y-3 = -11x+33 $11x+3y-36 = 0$	
11x + 3y - 36 = 0	
Equation of normal	1
$y - y_1 = m(x - x_1)$	
$y-1=\frac{3}{11}(x-3)$	
$ \begin{array}{r} 11y - 11 = 3x - 9 \\ 3x - 11y + 2 = 0 \end{array} $	1
b) A manufacturer can sell x items at a price of Rs.(330-x) each. The cost of 04)4
producing x items is Rs. $x^2 + 10x + 12$. Determine the number of items to be sold	
Ans so that the manufacturer can make the maximum profit. Selling price of x items = $(330-x)x = 330x - x^2$	
Cost price of x items = $x^2 + 10x + 12$	
Profit = Selling price – Cost price	
Let $P = (330x - x^2) - (x^2 + 10x + 12)$	
$= 330x - x^2 - x^2 - 10x - 12$ Page No.06/16	



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Q. No.	Sub Q. N.	Answer	Marking Scheme
3.	b)	$P = 320x - 2x^2 - 12$	1
J.			
		$\therefore \frac{dP}{dx} = 320 - 4x$	1
		Put $\frac{dP}{dx} = 0$	
		320 - 4x = 0	1
		$\therefore x = 80$	
		$\frac{d^2P}{dx^2} = -4 < 0$	1
		For maximum profit manufacturer can sell 80 items.	
	c)	If $x^y = e^{x-y}$ then prove that $\frac{dy}{dx} = \frac{\log x}{(1 + \log x)^2}$	04
	Ans	$x^{y} = e^{x-y}$	
		$\log x^{y} = \log e^{x-y}$	1/2
		$y\log x = (x-y)\log e$	1/2
		$y \log x = x - y$	
		$y \log x + y = x$	
		$y(\log x + 1) = x$	
		$y = \frac{x}{1 + \log x}$	1
		$\frac{dy}{dx} = \frac{(1+\log x)(1) - x\left(\frac{1}{x}\right)}{\left(1+\log x\right)^2}$	
		$\frac{dy}{dx} = \frac{(x)}{(1+\log x)^2}$	1
		$\frac{dy}{dx} = \frac{1 + \log x - 1}{\left(1 + \log x\right)^2}$	
			1
		$\frac{dy}{dx} = \frac{\log x}{\left(1 + \log x\right)^2}$	1
	d)	Evaluate $\int \frac{dx}{2x + x \cdot \log x}$	04
	Ans	$I = \int \frac{dx}{2x + x \cdot \log x}$	
		$=\int \frac{dx}{x(2+\log x)}$	1/2
		$\int x(2+\log x)$	/ -



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3.	d)	Put $2 + \log x = t$ OR Put $\log x = t$	1/2
		$\frac{1}{x}dx = dt$ $\frac{1}{x}dx = dt$	1
		$\therefore I = \int \frac{dt}{t} \qquad \qquad \therefore I = \int \frac{dt}{2+t}$	1/2
			1
		$= \log t + c \qquad \qquad = \log(2 + \log x) + c$	1/2
		$= \log(2 + \log x) + c \qquad \qquad = \log(2 + \log x) + c$	
4.		Solve any <u>THREE</u> of the following:	12
	a)	Evaluate: $\int \frac{dx}{x^2 + 4x + 25}$	04
	Ans	$I = \int \frac{dx}{x^2 + 4x + 25}$	
		$T.T. = \left(\frac{1}{2} \times Coeff.of.x\right)^2 = \left(\frac{1}{2} \times 4\right)^2 = 4$	1
		$x^{2} + 4x + 25 = x^{2} + 4x + 4 - 4 + 25$ $OR I = \int \frac{dx}{x^{2} + 4x - 4 + 4 + 25}$	1
		$= (x+2)^2 + 21 = (x+2)^2 + (\sqrt{21})^2$	
		$\therefore I = \int \frac{dx}{\left(x+2\right)^2 + \left(\sqrt{21}\right)^2}$	1
		$= \frac{1}{\sqrt{21}} \tan^{-1} \left(\frac{x+2}{\sqrt{21}} \right) + c$	1
	b)	Evaluate $\int \frac{dx}{2+3\cos 2x}$	04
	Ans	$I = \int \frac{dx}{2 + 3\cos 2x}$	
		Put $t = \tan x$, $\cos 2x = \frac{1 - t^2}{1 + t^2}$ and $dx = \frac{dt}{1 + t^2}$	1
		$\therefore I = \int \frac{\frac{dt}{1+t^2}}{2+3\left(\frac{1-t^2}{1+t^2}\right)}$	
		$= \int \frac{\frac{dt}{1+t^2}}{\frac{2(1+t^2)+3(1-t^2)}{1+t^2}}$ $= \int \frac{dt}{2+2t^2+3-3t^2}$	
		$= \int \frac{dt}{2 + 2t^2 + 3 - 3t^2}$	1/2
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4.	b)	$=\int \frac{dt}{5-t^2}$	
		$=\int \frac{dt}{\left(\sqrt{5}\right)^2 - t^2}$	1
		$= \frac{1}{2\left(\sqrt{5}\right)}\log\left(\frac{\sqrt{5}+t}{\sqrt{5}-t}\right) + c$	1
		$= \frac{1}{2(\sqrt{5})} \log \left(\frac{\sqrt{5} + \tan x}{\sqrt{5} - \tan x} \right) + c$	1/2
	c)	Evaluate $\int x \cdot \tan^{-1} x dx$	04
	Ans	$\int x \cdot \tan^{-1} x dx$	
		$= \int \tan^{-1} x . x dx$	
		$= \tan^{-1} x \int x dx - \int \left(\int x dx \frac{d(\tan^{-1} x)}{dx} \right) dx$	1
		$= \tan^{-1} x \left(\frac{x^2}{2}\right) - \int \left(\frac{x^2}{2}\right) \frac{1}{1+x^2} dx$	1
		$= \frac{x^2 \tan^{-1} x}{2} - \frac{1}{2} \int \frac{x^2}{1+x^2} dx$	
		$= \frac{x^2 \tan^{-1} x}{2} - \frac{1}{2} \int \frac{1 + x^2 - 1}{1 + x^2} dx$	
		$= \frac{x^2 \tan^{-1} x}{2} - \frac{1}{2} \int \left(1 - \frac{1}{1 + x^2} \right) dx$	1
		$= \frac{x^2 \tan^{-1} x}{2} - \frac{1}{2} \left(x - \tan^{-1} x \right) + c$	1
			04
	d)	Evaluate $\int \frac{x^2 + 1}{(x+1)(x+2)(x-3)} dx$	V T
	Ans	$\frac{x^2+1}{(x+1)(x+2)(x-3)} = \frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{x-3}$	1/2
		$\therefore x^2 + 1 = A(x+2)(x-3) + B(x+1)(x-3) + C(x+1)(x+2)$	
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		ame. Applied Mathematics Model Answer Subject Code.	
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4.	d)	Put $x = -1$: $A = \frac{-1}{2}$	1/2
		Put $x = -2$ $\therefore B = 1$	1/2
		Put $x = 3$ $\therefore C = \frac{1}{2}$	1/2
		$\therefore \int \frac{x^2 + 1}{(x+1)(x+2)(x-3)} dx = \int \frac{-1}{2} \frac{1}{x+1} + \frac{1}{x+2} + \frac{\frac{1}{2}}{x-3} dx$	1/2
		$\therefore \int \frac{x^2 + 1}{(x+1)(x+2)(x-3)} dx = \frac{-1}{2} \log(x+1) + \log(x+2) + \frac{1}{2} \log(x-3) + c$	1/2+1/2+1/2
		π/ ₂	04
		Evaluate $\int_{0}^{\pi/2} \frac{dx}{1 + \sqrt[3]{\tan x}}$	
		$\int_{0}^{\pi/2} \frac{dx}{1 + \sqrt[3]{\tan x}}$	
		$=\int_{0}^{\pi/2} \frac{dx}{1+\sqrt[3]{\frac{\sin x}{\cos x}}}$	
		$= \int_{0}^{\pi/2} \frac{dx}{1 + \frac{\sqrt[3]{\sin x}}{\sqrt[3]{\cos x}}}$	
		$=\int_{0}^{\pi/2} \frac{dx}{\sqrt[3]{\cos x} + \sqrt[3]{\sin x}}$	
		$I = \int_{0}^{\pi/2} \frac{\sqrt[3]{\cos x}}{\sqrt[3]{\cos x} + \sqrt[3]{\sin x}} dx \qquad(1)$	1
		$= \int_{0}^{\pi/2} \frac{\sqrt[3]{\cos\left(\frac{\pi}{2} - x\right)}}{\sqrt[3]{\cos\left(\frac{\pi}{2} - x\right)} + \sqrt[3]{\sin\left(\frac{\pi}{2} - x\right)}} dx $	
		$I = \int_{0}^{\pi/2} \frac{\sqrt[3]{\sin x}}{\sqrt[3]{\sin x} + \sqrt[3]{\cos x}} dx \qquad(2)$	1
		Add (1) and (2)	



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Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	e)	$\frac{\pi}{2}$ $3\sqrt{2}$ $3\sqrt{2}$	
	ŕ	$I + I = \int_{0}^{\pi/2} \frac{\sqrt[3]{\cos x}}{\sqrt[3]{\cos x} + \sqrt[3]{\sin x}} dx + \int_{0}^{\pi/2} \frac{\sqrt[3]{\sin x}}{\sqrt[3]{\sin x} + \sqrt[3]{\cos x}} dx$	
		$2I = \int_{-\frac{3}{4}}^{\frac{\pi}{2}} \frac{\sqrt[3]{\cos x} + \sqrt[3]{\sin x}}{\sqrt[3]{\cos x} + \sqrt[3]{\sin x}} dx$	
		$2I = \int_{0}^{\pi/2} 1 dx$	1
		$2I = [x]_0^{/2}$	
		$2I = \frac{\pi}{2}$	
		$\therefore I = \frac{\pi}{4}$	1
5.		Solve any <u>TWO</u> of the following:	12
	a)	Find the volume of the solid generated by revolving the ellipse	
		$\frac{x^2}{9} + \frac{y^2}{4} = 1$ about the x-axis	06
	Ans	$\frac{x^2}{9} + \frac{y^2}{4} = 1$	
		$\frac{x^2}{9} + \frac{y^2}{4} = 1 \text{ about the } x\text{-axis}$ $\frac{x^2}{9} + \frac{y^2}{4} = 1$ $\therefore \frac{y^2}{4} = 1 - \frac{x^2}{9}$ $\therefore y^2 = \frac{4}{9} (9 - x^2)$	
		$\therefore y^2 = \frac{4}{9} \left(9 - x^2 \right)$	1
		Volume of the solid generated by revolving the ellipse about the x-axis is given by	
		$V = \pi \int_{x=-3}^{x=3} y^2 dx$	
		$=\pi \int_{-3}^{3} \frac{4}{9} (9-x^2) dx$	1
		$=2\pi\int_{0}^{3}\frac{4}{9}(9-x^{2})dx \therefore \text{ Function is even}$	1
		$=2\pi \frac{4}{9} \left(9x - \frac{x^3}{3}\right)_0^3$	1
		$= \frac{8\pi}{9} \left[\left(9(3) - \frac{(3)^3}{3} \right) - 0 \right]$	1
		$=\frac{8\pi}{9}(27-9)$	
		$=16\pi$	1



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5.		Solve the following.	06
3.	b) (i)	Form the differential equation by eliminating the arbitrary constants if $y^2 = 4ax$	
	Ans	$y^2 = 4ax \qquad(1)$	03
		$2y\frac{dy}{dx} = 4a \qquad(2)$	1
		ta.	1
		Put (2) in (1)	
		$\therefore y^2 = 2y \frac{dy}{dx} x$	1
		$\therefore y^2 = 2y \frac{dy}{dx} x$ $\therefore y = 2x \frac{dy}{dx}$	
		$\therefore 2x \frac{dy}{dx} - y = 0$	1
		$\int_{-\infty}^{\infty} dx dx$	
	(ii)	Solve $(1+x^2)dy - (1+y^2)dx = 0$	03
	Ans	$(1+x^2)dy - (1+y^2)dx = 0$	
		$\left(1+x^2\right)dy = \left(1+y^2\right)dx$	
		$\frac{dy}{1+y^2} = \frac{dx}{1+x^2}$	1
		$1+y^2 1+x^2$ ∴ Solution is,	1
			1
		$\int \frac{dy}{1+y^2} = \int \frac{dx}{1+x^2}$	1
		$\tan^{-1} y = \tan^{-1} x + c$	
	c)	A resistance of 100Ω and inductance of 0.1 henries are connected in series	06
		with a battary of 20 volts. find the current in the circuit at any instant, if the	06
		relation between L,R and E is $L\frac{di}{dt} + Ri = E$	
	Ans	$L\frac{di}{dt} + Ri = E$	4.7
			1/2
		$\therefore \frac{dt}{dt} + \frac{R}{L}i = \frac{L}{L}$ Comparing with $\frac{dy}{dx} + Py = Q$	
		$\therefore \frac{di}{dt} + \frac{R}{L}i = \frac{E}{L} \qquad \text{Comparing with } \frac{dy}{dx} + Py = Q$ $\therefore P = \frac{R}{L} \text{ and } Q = \frac{E}{L}$	
		$IF = e^{\int \frac{R}{L} dt} = e^{\frac{R}{L}t}$	1
		$IF = e^{tL} = e^{tL}$ $\therefore \text{ Solution is } i \cdot IF = \int Q \cdot IF dt + c$	
			1
		$i \cdot e^{\frac{R}{L}t} = \int \frac{E}{L} e^{\frac{R}{L}t} dt + c$	
		Paga No.	10/17



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Q. Sub No. O. N. Answer	Marking
No. Q. N.	Scheme
5. c) $i \cdot e^{\frac{R}{L}t} = \frac{E}{L} \frac{e^{\frac{R}{L}t}}{\frac{R}{L}} + c$	1
5. $i \cdot e^{\frac{R}{L}t} = \frac{E}{L} \frac{e^{\frac{R}{L}t}}{\frac{R}{L}} + c$ $i \cdot e^{\frac{R}{L}t} = \frac{E}{R} e^{\frac{R}{L}t} + c$	1/2
Initially at $t = 0$, $i = 0$ $\therefore c = \frac{-E}{R}$ $\therefore \frac{R}{I} = \frac{R}{I} + \left(-E\right)$	
$i \cdot i \cdot e^{\frac{R}{L}t} = \frac{E}{R} e^{\frac{R}{L}t} + \left(\frac{-E}{R}\right)$ $i = \frac{E}{R} \left(1 - e^{-\frac{R}{L}t}\right)$	1
When R=100, L = 0.1, E= 20	
$i = \frac{20}{100} \left(1 - e^{-\frac{100}{0.1}t} \right)$ $i = 0.2 \left(1 - e^{-1000t} \right)$	1
6. Solve any <u>TWO</u> of the following:	12
a) Solve the following	06
(i) Find the approximate root of the equation $x^2 + x - 3 = 0$	03
in the interval (1,2) by using Bisection method(use two iterations)	
Ans $x^2 + x - 3 = 0$	
$f(x) = x^2 + x - 3$	
f(1) = -1 < 0	
f(2) = 3 > 0	1
root is in $(1,2)$	1
$\therefore x_1 = \frac{1+2}{2} = 1.5$	
$\therefore f(1.5) = 0.75 > 0$	
\therefore root is in $(1,1.5)$	
$\therefore x_2 = \frac{1+1.5}{2} = 1.25$	1
OR	
$x^2 + x - 3 = 0$	
$f(x) = x^2 + x - 3$	
f(1) = -1 < 0	
f(2) = 3 > 0	Dogo No 12/16



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Q. No.	Sub Q. N.	Answer	Marking Scheme
6.	(a)(i)	root is in $(1,2)$	1
		a b $x = \frac{a+b}{2}$ $f(x)$	
		1 2 1.5 0.75	1
		1 1.5 1.25	1
	(ii)	Solve the following system of the equations by using Gauss elimination method	03
		x + y + z = 6, $2x - 3y + 3z = 5$, $3x + 2y - z = 4$	
	Ans	x + y + z = 6	
		2x - 3y + 3z = 5	
		3x + 2y - z = 4	
		2x + 2y + 2z = 12 3x + 3y + 3z = 18	
		$2x-3y+3z=5 \qquad \text{and} \qquad 3x+2y-z=4$	
		5y - z = 7 $y + 4z = 14$	1
		20y - 4z = 28	
		y + 4z = 14	
		$\frac{+}{21y} = 42$	1/2
		21y - 42	, 2
		$\therefore y = 2$	1/2
		z = 3	1/2
		x = 1	1/2
		∴ Solution is {1,2,3}	
	b)	Solve the following system of equations by using Gauss Seidal method	06
		(use four iterations) correct upto 3 places of decimals.	
		x+7y-3z=-22, $5x-2y+3z=18$, $2x-y+6z=22$	
		Paga No 1/	4/1.6



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6.	b)Ans	5x-2y+3z=18, $x+7y-3z=-22$, $2x-y+6z=22$	1
		$x = \frac{1}{5}(18 + 2y - 3z)$	
		$y = \frac{1}{7} \left(-22 - x + 3z \right)$	1
		$z = \frac{1}{6}(22 - 2x + y)$	
		Starting with $x_0 = y_0 = z_0 = 0$	
		$x_1 = 3.6$	
		$y_1 = -3.657$	1
		$z_1 = 1.857$	
		1.022	
		$x_2 = 1.023$	1
		$y_2 = -2.493$ $z_2 = 2.910$	
		$z_2 - 2.510$	
		$x_3 = 0.857$	
		$y_3 = -2.018$	1
		$z_3 = 3.045$	
		$x_4 = 0.966$	
		$y_4 = -1.976$	1
		$z_4 = 3.015$	
	c)	Using Newton-Raphson method find the approximate root of the equation	
		correct upto 3 places of decimals. $x^3 - 2x - 5 = 0$ (Use four iterations)	06
	Ans	$f(x) = x^3 - 2x - 5$	
		f(2) = -1 < 0	
		f(3) = 16 > 0	
		Root is in $(2,3)$	1
		$f'(x) = 3x^2 - 2$	1
		Initial root $x_0=2$	
		$\therefore f'(2) = 10$	
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6.	c)	$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2 - \frac{f(2)}{f'(2)} = 2.1$	1
		$x_2 = 2.1 - \frac{f(2.1)}{f(2.1)} = 2.095$	1
		$x_3 = 2.095 - \frac{f(2.095)}{f'(2.095)} = 2.095$	1
		$x_4 = 2.095 - \frac{f(2.095)}{f'(2.095)} = 2.095$	1
		OR	
		$f(x) = x^3 - 2x - 5$	
		f(2) = -1 < 0	
		f(3) = 16 > 0	
		Root is in $(2,3)$	
		$f'(x) = 3x^2 - 2$	
		Initial root $x_0=2$	
		$x_{i} = \frac{xf'(x) - f(x)}{f'(x)}$	
		$=\frac{x(3x^2-2)-(x^3-2x-5)}{3x^2-2}$	
		$=\frac{3x^3-2x-x^3+2x+5}{3x^2-2}$	
		$=\frac{2x^3+5}{3x^2-2}$	2
		$x_1 = 2.1$	1
		$x_2 = 2.095$	1
		$x_3 = 2.095$	1
		$x_4 = 2.095$	1
		Important Note	
		In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.	
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