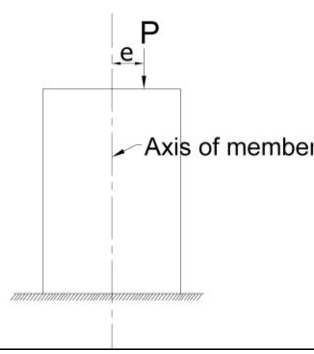
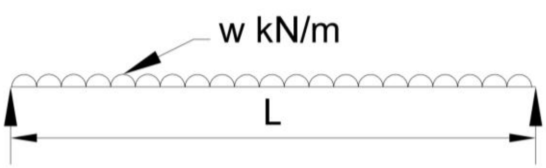
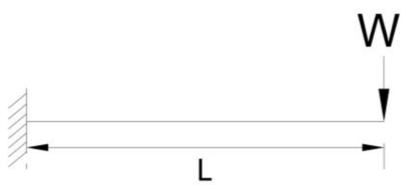
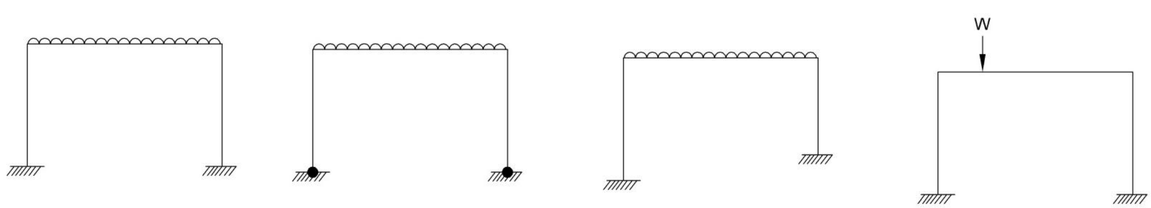
WINTER– 17 EXAMINATION

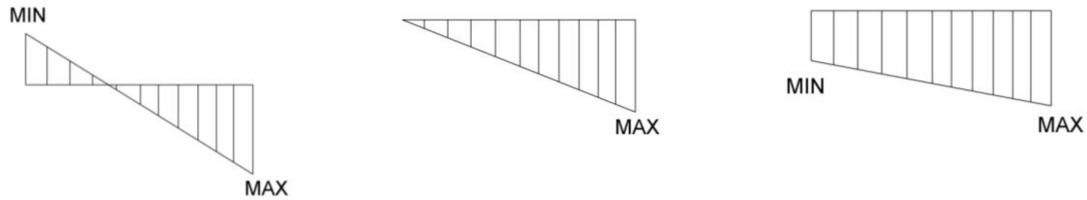
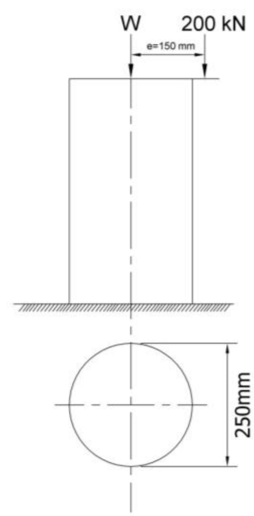
Subject Name: Theory of Structures **Model Answer** Subject Code: 17422

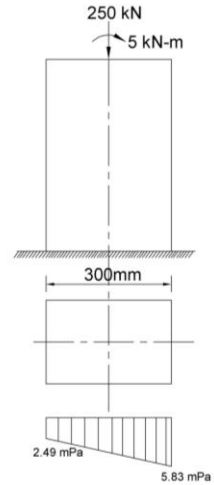
Important Instructions to examiners:

1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
3) The language errors such as grammatical, spelling errors should not be given more Importance (Not applicable for subject English and Communication Skills.
4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answers and model answer.
6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
7) For programming language papers, credit may be given to any other program based on equivalent concept.

Q. No.	Sub Q. N.	Answer	Marking Scheme
Q.1	(A)i) Ans	Define "Eccentric load with neat sketch. Eccentric loading: It is defined as load whose line of action does not coincide with the longitudinal axis of the member.  	01 Mark 01 Mark
Q.1	(A)ii) Ans	Write the values of maximum slope and deflection in case of simply supported beam with u.d.L. over the entire span in terms of EI.  Maximum slope = (w x L^3) / (24 x EI) at supports. -- Maximum deflection = (5 x w x L^4) / (384 x EI) at midspan. --------------------------------------	 01 Mark 01 Mark
Q.1	(A)iii) Ans	Write the differential equation for slope and deflection and state terms used in the equation. In the theory of simple bending, curvature of beam is expressed as – 1 / R = d^2y/dx^2 For bending equation, 1 / R = M / EI	

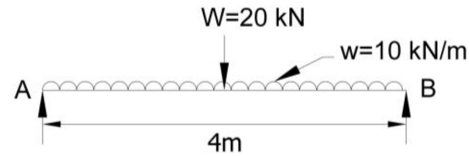
		$M / EI = d^2y/dx^2$ $EI(d^2y/dx^2) = M_x$ --- differential equation Where, E = Modulus of elasticity. I = Moment of inertia of C/S. M_x = Bending moment at the section X-X in beam.	01 Mark 01 Mark
Q.1	(A)iv) Ans	State values of maximum slope and deflection for cantilever beam of span L carrying a point load at free end with meaning of each term.  Maximum slope = $(WL^2) / 2EI$ at free end -- Maximum deflection = $(WL^3) / 3EI$ at free end --	01 Mark 01 Mark
Q.1	(A)v) Ans	State any two disadvantage of fixed beam. i. A little sinking of one support over the other induces additional moment at each end. ii. Due to end fixity, temperature stresses are induced due to variation in temperature. iii. Extra care has to be taken to achieve perfect fixity at ends. iv. Frequent fluctuations in loading (moving loads) are likely to disturb end fixity. v. Practically it is difficult to produce 100% fixity.	Any Two 01 Mark Each
Q.1	(A)vi) Ans	With sketch state the different types of portal frame. i. Symmetrical portal frame (Non sway type) ii. Unsymmetrical portal frame (Sway type) 	01 Mark each
Q.1	(A)vii) Ans	Define carry over moment and carry over factor. i. Carry over moment: It is defined as the moment induced at the far fixed end of beam by the action of the moment applied at the near simply supported or hinged end. ii. Carry over factor: It is the ratio of moment induced at far end to the moment applied at near end without displacing it.	01 Mark 01 Mark
Q.1	(A)viii) Ans	List out different types of roof trusses any four. i. King post truss ii. Queen post truss iii. Simple fink truss iv. Compound fink truss v. Fan truss vi. Pratt truss vii. Howe truss viii. North light truss	Any Four ½ Mark each
Q.1	(B)i) Ans	Define core of a section and state middle third rule. i. Core of a section: It is defined as the region or area within which if load is applied, produces only compressive resultant stress. ii. Middle third rule: In case of rectangular cross section, if the load is applied at location along the middle third part of both mutually perpendicular axes then the stresses produced are wholly of compressive nature.	02 Marks 02 Marks

Q.1	(B)ii) Ans	Draw resultant stress diagram for $6_0 < 6_b$, $6_0 = 6_b$, $6_0 > 6_b$. i) $6_0 < 6_b$ ii) $6_0 = 6_b$ iii) $6_0 > 6_b$  Where, 6_0 = Direct stress and 6_b = Bending stress ---	01 Mark each for dia. 01 Mark
Q.1	(B)iii) Ans Ans	a) State the assumptions in the analysis of frame. 1. All joints in frame are pinned or hinged. 2. Loads are applied at joints only. 3. Self-weight of members of frame is neglected. 4. Only axial forces (tensile and compressive) are induced in the member. b) Define redundant frame and state its condition. Redundant frame: It is the frame which cannot be anaysed internally using basic equations of equilibrium ($\Sigma M_A = 0$, $\Sigma F_x = 0$ and $\Sigma F_y = 0$). Condition: $m > (2j – 3)$ where, m = Number of members and j = No. of joints.	½ mark for each 01 Mark 01 Mark
Q.2	a) Ans	A solid circular column of diameter 250 mm carries an axial load 'W' kN and a load of 200 kN at an eccentricity of 150mm. Calculate minimum value of 'W' so as to avoid the tensile stresses at base.  $A = (\pi \times 250^2) / 4 = 49087.38\ mm^2$ $M = P \times e = 200 \times 10^3 \times 150 = 3 \times 10^7$ N-mm $I = (\pi \times 250^4) / 64 = 1.917 \times 10^8\ mm^4$ y = D / 2 = 250 / 2 = 125 mm. $6_0 = (W + P) / A = (W + 200 \times 10^3) / 49087.38$ $6_b = (M \times y) / I = 3 \times 10^7 \times 125 / 1.917 \times 10^8$ $= 19.562\ N/mm^2$ For no tension, $6_0 = 6_b$ $(W + 200 \times 10^3) / 49087.38 = 19.562$ W = 760238.3 N = 760.24 kN.	 01 Mark 01 Mark 01 Mark 01 Mark

Q.2	b)	A rectangular column 300 mm wide and 200 mm thick carries an axial load of 250 kN and a clockwise moment of 5 kN m in plane bisecting 200 mm side, calculate resultant stresses induced at the base.	
	Ans	Axial load = P 250 kN = 250 x 10^3 N	
		B = 200 mm, d = 300 mm	
		A = 200 x 300 = 60000 mm^2	01 Mark
		I = 200 x 300^3 / 12 = 4.5 x 10^8 mm^4	
		y = d / 2 = 300 / 2 = 150 mm	
		M = 5 kN-m = 5 x 10^6 N-mm	
		6_0 = P / A = 250 x 10^3 / 60000 = 4.16 N/mm^2	01 Mark
		6_b = (M x y) / I = 5 x 10^6 x 150 / 4.5 x 10^8	01 Mark
		= 1.67 N/mm^2	
		6_{max} = 6_0 + 6_b = 4.16 + 1.67 = 5.83 N/mm^2	½ Mark
		6_{min} = 6_0 - 6_b = 4.16 - 1.67 = 2.49 N/mm^2	½ Mark
		 	
Q.2	c)	A masonry wall 10 m high, 3 m wide and 1.5m thick is subjected to a wind pressure of 1.2 kN/m^2. Find maximum and minimum intensity induced on the base if the unit weight of masonry is 22kN/m^3.	
	Ans	Area at base of wall = 3 x 1.5 = 4.5 m^2 Height of wall (h) = 10 m,	
		Unit weight of material (σ) = 22 kN/m^3 Weight of wall (W) = 22 x 4.5 x 10 = 990 kN.	
		6_0 = σh OR 6_0 = W / A	
		= 22 x 10 = 220 kN/m^2 = 990 / 4.5 = 220 kN/m^2 -------------------------	01 Mark
		I = 3 x 1.5^3 / 12 = 0.84375 m^4 y = 1.5 / 2 = 0.75 m ---	½ Mark
		Wind force (P) = Wind pressure x b x h	
		= 1.2 x 3 x 10 = 36 kN ---	½ Mark
		-	
		Moment @ base (M) = P x h/2	
		= 36 x 10 / 2 = 180 kN-m ---	½ Mark
		6_b = (M x y) / I = 180 x 0.75 / 0.84375 = 160 kN/m^2 --	½ Mark
		6_{max} = 6_0 + 6_b = 220 + 160 = 380 kN/m^2 ---	½ Mark
		6_{min} = 6_0 - 6_b = 220 - 160 = 60 kN/m^2 ---	½ Mark
Q.2	d)	A wooden cantilever beam of span 2.5 m has a cross section 130 mm wide and 240 mm deep. A load of 6 kN is acting at free end, calculate the deflection and slope at free end take E = 1 x 10^5 N/mm^2.	
	Ans	b = 130 mm, d = 240 mm, Span = 2.5 m.= 2500 mm	
		E = 1 x 10^5 N/mm^2 I = 130 x 240^3 / 12 = 1.4976 x 10^8 mm^4 W = 6 kN = 6000 N	01 Mark
		Slope at free end = WL^2 / 2EI	½ Mark
		= 6000 x 2500^2 / (2 x 1 x 10^5 x 1.4976 x 10^8) = 1.252 x 10^{-3} rad.	01 Mark
		Deflection at free end = WL^3 / 3EI	½ Mark
		= 6000 x 2500^3 / (3 x 1 x 10^5 x 1.4976 x 10^8) = 2.087 mm.	01 Mark

Q.2 e) A simply supported beam of span 4 m carries a central point load of 20 kN and u.d.L. of 10 kN/m over entire span. Find maximum slope and maximum deflection of the beam I_{xx}=2 x 10^8mm^4 E= 2 x 10^5N/mm^2.

Ans



EI = 2 x 10^8 x 2 x 10^5 = 4 x 10^{13} N-mm^2

Maximum slope = $(W \times L^2 / 16EI)_{P.L.} + (w \times L^3 / 24EI)_{U.D.L.}$ at supports ---------------------------- 01 Mark

= (20000 x 4000^2) / (16 x 4 x 10^{13}) + (10 x 4000^3) / (24 x 4 x 10^{13}).

= 5 x 10^{-4} + 6.66 x 10^{-4}

= 1.166 x 10^{-3} rad. -- 01 Mark

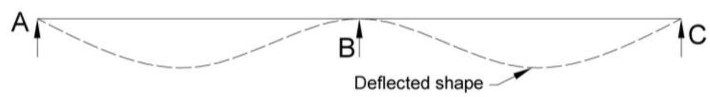
Maximum deflection = $(W \times L^3 / 48EI)_{P.L.} + (5 \times w \times L^4 / 384EI)_{U.D.L.}$ at mid-span ---------------- 01 Mark

= (20000 x 4000^3 / 48 x 4 x 10^{13}) + (5 x 10 x 4000^4 / 384 x 4 x 10^{13})

= 0.66 + 0.83 = 1.49 mm -- 01 Mark

Q.2 f) State the effect of continuity on the continuous beam. Explain with sketch.

Ans



02 Marks

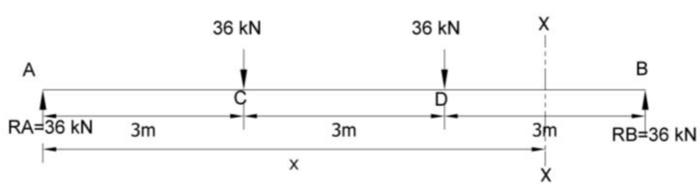
Effects of continuity are as follows.

i. Produces support moment of hogging nature.
ii. Reduces bending moment along the span.
iii. Reduces deflection and increases load carrying capacity.
iv. Sagging moment occurs at mod span.

02 Marks

Q.3 a) A simply supported beam of span 9 m carries two point loads of equal magnitude 36 kN at 3 m from both ends. Calculate values of integration constant and write Macaulay's slope and deflection equation.

Ans



Calculation of Reactions:

$\Sigma M_A = 0$

36 x 3 + 36 x 6 – R_B x 9 = 0

R_B = (108 + 216) / 9

= 36 kN.

R_A = 72 – 36 = 36 kN.

OR

Due to symmetry,

$R_A = R_B$ = 72 / 2

= 36 kN.

Taking section X-X at distance 'x' from A.

$M_x = 36 \times x - 36(x - 3) - 36(x - 6)$

$EId^2y/dx^2 = -Mx$

$= -36 \times x + 36(x - 3) + 36(x - 6)$

01 Mark

Integrating

$EIdy/dx = (-36 \times x^2)/2 + [36(x - 3)^2]/2 + [36(x - 6)^2]/2 + C_1$ ------------- 1

Integrating

$EIy = (-18 \times x^3)/3 + [18(x-3)^3]/3 + [18(x-6)^3]/3 + C_1x + C_2$ ------------- 2

At $x = 0$; $y = 0$ in eq^n. 2

$0 = 0 + C_2$

$C_2 = 0$

At $x = 9$; $y = 0$ in eq^n. 2

$0 = (-18 \times 9^3)/3 + [18(9-3)^3]/3 + [18(9-6)^3]/3 + C_1 \times 9 + 0$

$C_1 = 324$

Hence $C_1 = 324$ and $C_2 = 0$ — 01 Mark

Slope equation:

$dy/dx = 1/EI[(-18 \times x^2) + 18(x-3)^2 + 18(x-6)^2 + 324]$ — 01 Mark

Deflection equation:

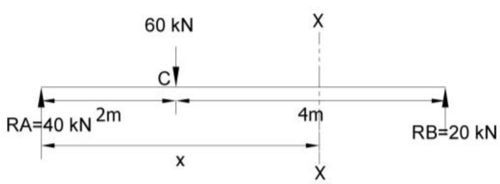
$y = 1/EI[(-6 \times x^3) + 6(x-3)^3 + 6(x-6)^3] + 324x$ — 01 Mark

Q.3 b)

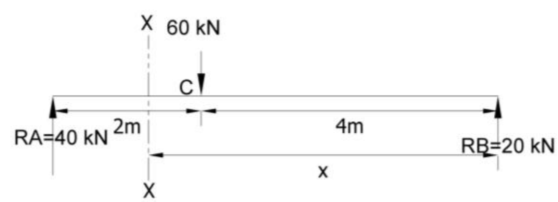
A simply supported beam of 6 m span carries a point load of 60 kN at 2m from left support.

Calculate deflection below point load in terms of EI use Macaulay's method.

Ans



OR



Reactions:

$\Sigma M_A = 0$

$= 60 \times 2 - R_B \times 6$

$R_B = 120 / 6 = 20$ kN. — 01 Mark

$R_A = 60 - 230 = 40$ kN.

Taking section X-X at distance 'x' from A	Taking section X-X at distance 'x' from B
$M_x = 40 \times x - 60(x-2)$	$M_x = 20 \times x - 60(x-4)$
$EId^2y/dx^2 = -Mx$	$EId^2y/dx^2 = -Mx$
$= -40 \times x + 60(x-2)$	$= -20 \times x + 60(x-4)$
Integrating	Integrating
$EIdy/dx = (-40 \times x^2)/2 + [60(x-2)^2]/2 + C_1$	$EIdy/dx = (-20 \times x^2)/2 + [60(x-4)^2]/2 + C_1$
Integrating	Integrating
$EIy = (-20 \times x^3)/3 + [30(x-2)^3]/3 + C_1x + C_2$	$EIy = (-10 \times x^3)/3 + [30(x-4)^3]/3 + C_1x + C_2$
At $x = 0$; $y = 0$ in EIy eq^n.	At $x = 0$; $y = 0$ in EIy eq^n.
$0 = 0 + C_2$	$0 = 0 + C_2$
$C_2 = 0$	$C_2 = 0$
At $x = 6$; $y = 0$ in EIy eq^n.	At $x = 6$; $y = 0$ in EIy eq^n.
$0 = (-20 \times 6^3)/3 + [10(9-2)^3] + C_1 \times 6 + 0$	$0 = (-10 \times 6^3)/3 + [10(9-4)^3] + C_1 \times 6 + 0$
$C_1 = 133.33$	$C_1 = 106.67$
Hence $C_1 = 133.33$ and $C_2 = 0$	Hence $C_1 = 106.67$ and $C_2 = 0$

01 Mark

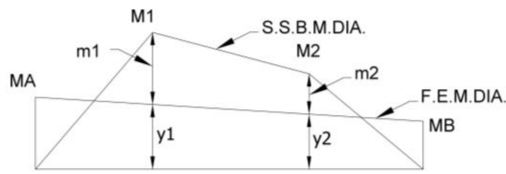
Deflection equation- y =(1/EI)[(- 20 x x^3)/3+10(x – 2)^3+133.33x] For deflection under load Put x = 2 in eq^n. y_c = (1/EI)[(- 20 x 2^3)/3+ 0 +133.33 x 2] = 213.33 / EI	Deflection equation- y =(1/EI)[(- 10 x x^3)/3+10(x – 4)^3+106.67x] For deflection under load Put x = 4 in eq^n. y_c = (1/EI)[(- 10 x 4^3)/3+ 0 +106.67 x 4] = 213.33 / EI

01 Mark

01 Mark

Q.3 c) State how B. M. is find out for a fixed beam using super position theorem. Explain it with sketch.

Ans After calculating fixed end moments and simply supported bending moments, draw fixed end moment diagram and superimpose simply supported bending moment diagram over it as shown in fig below.



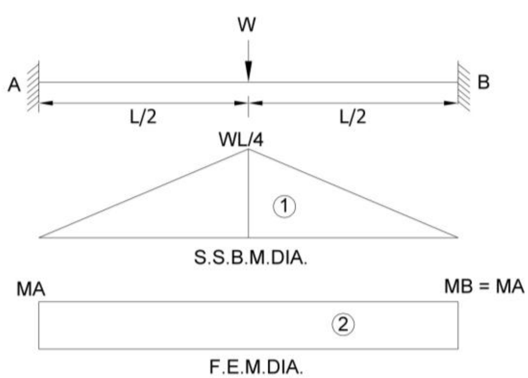
02 Marks

Let net bending moment at 1 and 2 are m_1 and m_2 respectively.
Calculate y_1 and y_2 by interpolation.
Then $m_1 = M_1 – y_1$ and $m_2 = M_2 – y_2$

02 Marks

Q.3 d) Using first principle find fixed end moment for a fixed beam carrying point load at mid span.

Ans



01 Mark

Due to symmetry, $M_A = M_B$
Area of S. S. B. M. Dia. = a_1 = 0.5 x L x WL/4 = WL^2 / 8 — 01 Mark
Area of F. E. M. Dia. = M_A x L
Area of simply supported bending moment diagram = Area of fixed end moment diagram — 01 Mark

$$a_1 = a_2$$

WL^2 / 8 = M_A x L
Hence M_A = WL / 8 And M_B = WL / 8 — 01 Mark

Q.3 e) Explain imperfect and perfect frame in detail.

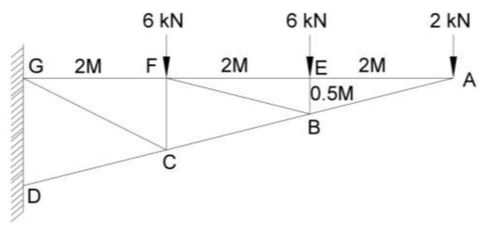
Ans **Perfect frame**: It is the simple frame in which number of joints (j) and number of members (m) satisfies the equation m = 2j – 3. Such frames are internally determinate i.e. can be analysed by using basic equations of equilibrium ($\Sigma M_A = 0$, $\Sigma F_x = 0$ and $\Sigma F_y = 0$). — 02 Marks

Imperfect frame: It is the simple frame in which number of joints (j) and number of members (m) does not satisfy the equation m = 2j – 3. Such frames are internally indeterminate/redundant or deficient. — 02 Marks

If m > 2j – 3; then frame is called as indeterminate/redundant frame and cannot be analysed by using basic equations of equilibrium ($\Sigma M_A = 0$, $\Sigma F_x = 0$ and $\Sigma F_y = 0$).

If m < 2j – 3; then frame is called as deficient frame and it is unstable frame.

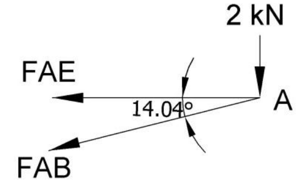
Q.3 f) Determine the forces along with nature in the members AB,AE, EB and EF for frame subjected to a load as shown is Fig. using method of joints.



Ans

Consider joint A.

$\Theta = \tan^{-1}(0.5/2)$
$= 14.04^0$



02 Marks

Assuming F_{AE} and F_{AB} both tensile.

$\Sigma F_y = 0$; $2 + F_{AB}\,\text{Sin}14.04 = 0$

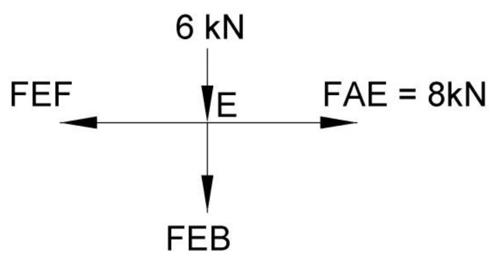
$F_{AB} = -2/\,\text{Sin}14.04 = -8.24$ i.e 8.24 kN (Compressive)

$\Sigma F_x = 0$; $-F_{AE} - F_{AB}\cos 14.04 = 0$

$-F_{AE} - (-8.24)\cos 14.04 = 0$

$F_{AE} = 8.0$ kN (Tensile)

Consider joint E.



02 Marks

Assuming F_{EF} and F_{EB} both tensile.

$\Sigma F_x = 0$; $-F_{EF} + 8.0 = 0$

$F_{EF} = 8.0$ kN (Tensile)

$\Sigma F_y = 0$; $-6.0 - F_{EB} = 0$

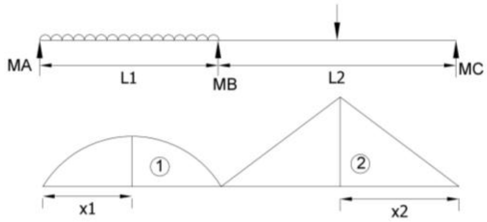
$F_{EB} = -6.0$ i.e 6.0 kN (Compressive)

Member	Force	Nature
AB	8.24 kN	Compressive
AE	8.0 kN	Tensile
EB	6.0 kN	Compressive
EF	8.0 kN	Tensile

Q.4 a) State Clapeyron's theorem and also write the Clapeyron's three moment theorem for beam with different moment of inertial giving meaning of each term.

Ans. Clapeyrons theorem: For two span continuous beam having uniform moment of inertia supported at A, B, and C and subjected to any external loading, the support moments M_A, M_B and M_C at the supports A, B and C respectively are given by the relation,

$M_A \times L_1 + 2M_B(L_1 + L_2) + M_C \times L_2 = -[(6 \times a_1 \times x_1/L_1) + (6 \times a_2 \times x_2/L_2)]$ — 02 Marks



If moment of inertia is not constant then Clapeyrons theorem can be stated in form of following equation.

$M_A \times (L_1/I_1) + 2M_B[(L_1/I_1) + (L_2/I_2)] + M_C \times (L_2/I_2) = -[(6 \times a_1 \times x_1/L_1I_1) + (6 \times a_2 \times x_2/L_2I_2)]$ — 01 Mark

Where,

L_1 & L_2 are length of span AB & BC resp.

I_1 & I_2 are Moment of inertia of span AB & BC resp.

a_1 & a_2 are area of simply supported BMD of span AB & BC resp. — 01 Mark

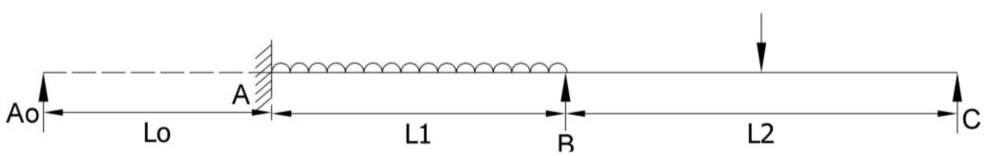
x_1 & x_2 are distances of centroid of simply supported BMD from A & C resp.

Q.4 b) Explain the concept of imaginary zero span in case of Clapeyron's theorem.

Ans When the ends of continuous beam are fixed, then an imaginary span is considered to the left or right of the fixed support as the case may be and Clapeyrons theorem is applied to the imaginary span and its adjescent span as per regular procedure. — 02 Marks

If left end is fixed then consider imaginary span left of this support and If right end is fixed then consider imaginary span on right side of that support.

Clapeyrons theorem is applied as below. — 01 Mark



A_0-A is imaginary span left to fixed end A.

For span A_0-A and AB

$M_{A0} \times L_0 + 2M_A(L_0 + L_1) + M_B \times L_1 = -[(6 \times a_0 \times x_0/L_0) + (6 \times a_1 \times x_1/L_1)]$ — 01 Mark

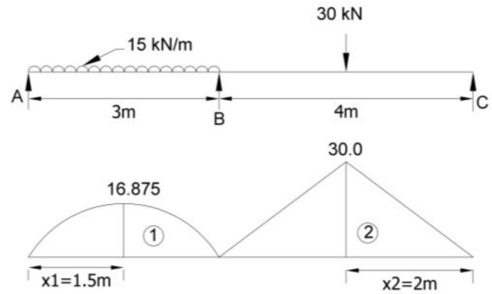
$0 + 2M_A(L_1) + M_B \times L_1 = -[0 + (6 \times a_1 \times x_1/L_1)]$

Where M_{A0}, L_0 and x_0 are terms related to imaginary span.

Q.4 c) A beam ABC is supported at A, B and C span AB and BC are of lengths 3 m and 4 m respectively. AB carries a u.d.L. of 15 kN/m over entire span and BC carries central point load of 30 kN. Calculate support moment at B using three moment theorem.

01 Mark

Ans



01 Mark

01 Mark

$M_1 = 15 \times 3^2/8 = 16.875$ kN-m $a_1 = 2 \times 3 \times 16.875 / 3 = 33.75$ $x_1 = 3/2 = 1.5$ m

$M_2 = 30 \times 4/4 = 30.0$ kN-m $a_2 = 0.5 \times 4 \times 30 = 60.0$ $x_2 = 4/2 = 2$ m

Using three moment theorem; — 01 Mark

$M_A \times l_1 + 2M_B (l_1 + l_2) + M_C \times l_2 = -[(6 \times a_1 \times x_1/l_1) + 6 \times a_2 \times x_2/l_2)]$

$M_A = M_C = 0$ (End simple supports)

$2M_B(3 + 4) = -[(6 \times 33.75 \times 1.5/3) + 6 \times 60.0 \times 2.0/4)]$

$14.0M_B = -101.25 - 180$

$M_B = -281.25/14 = -20.09$ i.e. 20.09 kN-m Hogging

Q.4 d) Define stiffness of beam and state stiffness factor for beam with far end fixed and simply supported end.

Ans **Stiffness of beam:** The moment required to produce unit rotation at the near end is called as stiffness of beam. — 02 Marks

Stiffness factor for beam with far end fixed = 4EI/L — 01 Mark

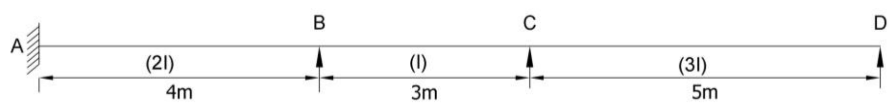
Stiffness factor for beam with far end simply supported = 3EI/L — 01 Mark

EI = Flexural rigidity of beam.

L = Length of beam.

Q.4 e) Determine distribution factors at continuity for a continuous beam ABCD which is fixed at A
and supported at B, C and D. Take AB =4 m, BC =3 m and CD = 5 m if M.I. for the spans is $I_{AB} = 2I$, $I_{BC} = I$, $I_{CD} = 3I$.

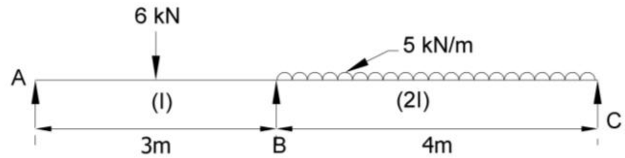
Ans



Joint	Member	Stiffness (k)	Σk	D.F. = k/Σk
B	BA	4 x 2EI/4 = 2EI	3EI	2EI/3EI = 0.67
	BC	3 x EI/3 =EI		EI/3EI = 0.33
C	CB	3 x EI/3 =EI	2.8EI	EI/2.8EI = 0.36
	CD	3 x 3EI/5 =1.8EI		1.8EI/2.8EI = 0.64

01 Mark for each

Q.4 f) Calculate support moments by moment distribution method for given continuous as shown
in fig.



Ans

$M_{AB} = -6 \times 3/8 = -2.25$ kN-m $\quad$ $M_{BA} = 6 \times 3/8 = 2.25$ kN-m

$M_{BC} = -5 \times 4^2/12 = -6.67$ kN-m $\quad$ $M_{BC} = 5 \times 4^2/12 = 6.67$ kN-m

Joint	Member	Stiffness (k)	Σk	D.F. = k/Σk
B	BA	3 x EI/3 = EI	2.5EI	EI/2.5EI = 0.4
	BC	3 x 2EI/4 =1.5EI		1.5EI/2.5EI = 0.6

01 Mark

Joint	A	B		C
Members	AB	BA	BC	CB
Dist^n. factor	1.0	0.4	0.6	1.0
F.E.M.	– 2.25	2.25	– 6.67	6.67
Balancing	2.25	1.768	2.652	– 6.67
Carry over		1.125	– 3.33	
Balancing		0.882	1.323	
Final moments	0.0	6.025	– 6.025	0.0

02 Marks

Support moment at B = MB = 6.025 kN-m (Hogging) — 01 Mark

Q.5 a) A circular chimney has external diameter 60% more than internal diameter. The height of chimney is 32 m and is subjected to a horizontal wind pressure of 1.75 kN/m^2. Find out the diameter of the chimney so as to avoid tension at the base of chimney and also draw stress distribution diagram. Unit wt. of chimney material is 18 kN/m^3 and C = 0.6.

Ans

Data: External diameter (D) = 1.6 x internal diameter (d)

Height of chimney (h) = 32 m

Horizontal wind pressure (p) = 1.75 kN/m^2

Unit weight of material (σ) = 18 kN/m^3 $\quad$ C = 0.6

$6_d = \sigma h = 18 \times 32 = 576$ kN/m^2 -- 01 Mark

Horizontal wind force (P) = p x h x D x C

= 1.75 x 32 x 1.6d x 0.6

= 53.76d -- 01 Mark

Moment about base (M) = P x h/2

= 53.76d x 32/2

= 860.16d ----------------- -------------- 01 Mark

$I = (\pi / 64)(D^4 - d^4)$

$= (\pi / 64)[(1.6d)^4 - d^4]$

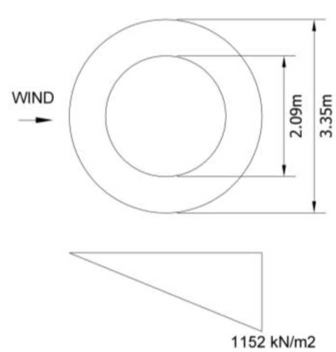
$= 0.273d^4$

$y_{max} = 0.8d$

$6_b = M \times y_{max}/I$

$= 860.16d \times 0.8d / 0.273d^4$

$= 2520.6 / d^2$ --- ------------ 01 Mark



01 Mark for stress dia.

For no tension;

$б_d = б_b$ -- 01 Mark

$576 = 2520.6 / d^2$

$d^2 = 4.38$

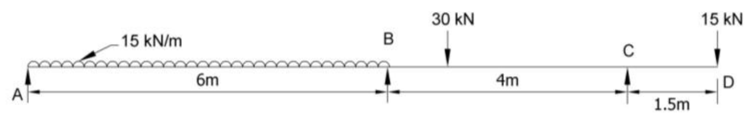
$d = 2.092$ m

$D = 1.6 \times 2.092 = 3.35$ m -- 01 Mark

$б_{max} = 2\ б_d = 2 \times 576 = 1152$ kN/m^2 --- 01 Mark

Q.5 b) A beam ABCD is supported at A, B and C span CD is having overhang AB = 6 m BC = 4 m and CD = 1.5 m span AB carries UDL of 15 kN/m over entire span and BC carries point load of30 kN at 1 m from support B and a point load of 15 kN acts at free end at D. Determine support moments using moment distribution method and draw BMD.

Ans



$M_{AB} = -15 \times 6^2/12 = -45.0$ kN-m $\quad M_{BA} = 15 \times 6^2/12 = 45.0$ kN-m — 02 marks

$M_{BC} = -30 \times 1 \times 3^2/4^2 = -16.875$ kN-m $\quad M_{BC} = 30 \times 1^2 \times 3/4^2 = 5.625$ kN-m

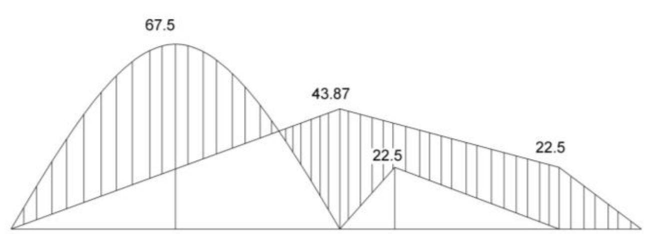
MCD = – 15 x 1.5 = – 22.5 kN-m

Joint	Member	Stiffness (k)	Σk	D.F. = k/Σk
B	BA	3 x EI/6 = 0.5EI	1.25EI	0.5EI/1.25EI = 0.4
	BC	3 x EI/4 =0.75EI		0.75EI/1.25EI = 0.6

02 Marks

Joint	A		B		C
Members	AB	BA	B	CB	CD
Dist^n. factor	1.0	0.4	0.6	1.0	0.0
F.E.M.	– 45.0	45.0	– 16.875	5.625	– 22.5
Balancing Carry over	45.30	–11.25 22.5	– 16.875 8.44	16.875	0.0
Balancing		–12.38	–18.56		
Final moments	0.0	43.87	– 43.87	22.5	–22.5

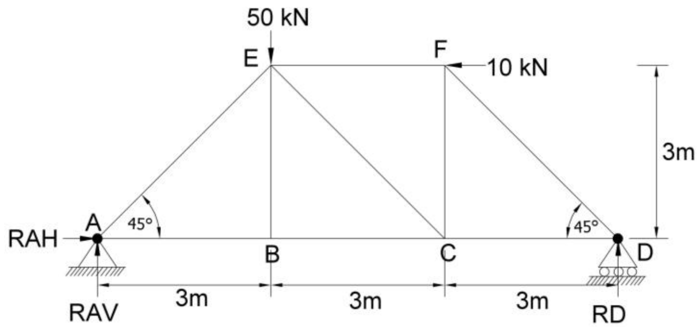
02 Marks



B. M. D.

02 Marks

Q.5 c) Using method of section find forces in members BC, BE and EF and EC for truss shown in Fig.State nature of forces tabulate results.



Ans

Reactions:

$\Sigma M_A = 0 = 50 \times 3 - 10 \times 3 - R_D \times 9$

$R_D = 13.33$ kN

$R_{AV} = 50 - 13.33 = 36.67$ kN

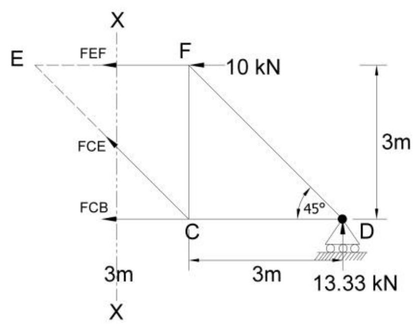
$\Sigma F_V = 0$

$R_{AH} - 10 = 0$

$R_{AH} = 10$ kN — 01 Mark

Taking section along EF, EC and BC

Assuming all forces Tensile



01 Mark

Taking moment @ C;

$-13.33 \times 3 - 10 \times 10 - F_{EF} \times 3 = 0$

$F_{EF} = -23.33$ i.e. 23.33 kN (Compressive)

Taking moment @ E;

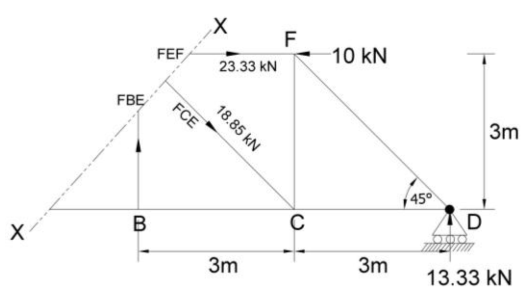
$F_{CB} \times 3 - 13.33 \times 6 = 0$

$F_{CB} = 26.67$ kN (Tensile) — 02 Marks

$\Sigma F_V = 0 = 13.33 + F_{CE} \sin 45$

$F_{CE} = -18.85$ i.e. 18.85 kN (Compressive)

Taking section along EF, EC, EB and BA



01 Mark

$\Sigma F_V = 0 = 13.33 - 18.85 \sin 45 + F_{BE}$

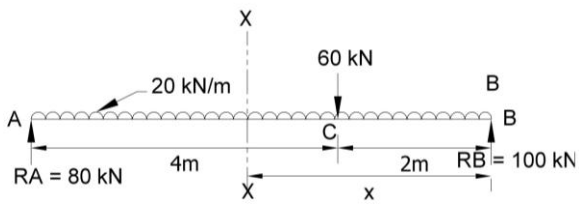
$F_{BE} = 0$ — 01 Mark

Member	Force	Nature
BC	26.67 kN	Tensile
BE	0	--
EF	23.33 kN	Compressive
EC	18.85 kN	Compressive

02 Marks

Q.6 a) A simply supported beam of 6 m span carries an u.d.l. of 20 kN/m over entire beam and a point load of 60 kN at 2 m from right hand support using Macaulay's method, locate the point of maximum deflection and find its value in terms of EI.

Ans



Reactions:

$\Sigma M_A = 0$

$60 \times 4 + 20 \times 6 \times 3 - R_B \times 6 = 0$

$R_B = (240 + 360) / 6$ — 01 Mark

$= 100$ kN.

$R_A = 20 \times 6 + 60 - 100 = 80$ kN.

Taking section X-X at distance 'x' from B.

$M_x = 100 \times x - 60(x - 2) - 20 \times x^2/2$ — 01 Mark

$EId^2y/dx^2 = - M_x$

$= - 100 \times x + 60(x - 2) + 20 \times x^2/2$

Integrating

$EIdy/dx = (- 100 \times x^2/2) + [60(x - 2)^2/2] + 10 \times x^3/3 + C_1$ ------------- 1 — 01 Mark

Integrating

$EIy = (- 50 \times x^3/3) + [30(x - 2)^3/3] + 10 \times x^4/12 + C_1x + C_2$ ------------- 2 — 01 Mark

At $x = 0$; $y = 0$ in eq^n. 2

$0 = 0 + C_2$

$C_2 = 0$

At $x = 6$; $y = 0$ in eq^n. 2

$0 = (- 50 \times 6^3/3) + [30(6 - 2)^3/3] + 10 \times 6^4/12 + C_16 + C_2$

$C_1 = 313.33$

Hence $C_1 = 313.33$ and $C_2 = 0$ — 01 Mark

Slope equation:

$dy/dx = 1/EI(- 50 \times x^2) + [30(x - 2)^2] + 10 \times x^3/3 + 313.33$ ------------- I

Deflection equation: — 01 Mark

$y = 1/EI(- 50 \times x^3/3) + [10(x - 2)^3] + 10 \times x^4/12 + 313.33x$ ------------- II

Deflection is maximum where slope changes the sign i.e. slope = 0

Maximum deflection will be in between A and C

Hence, $6 > x > 2$

Equating Equ^n I with zero.

$0 = 1/EI(- 50 \times x^2) + [30(x - 2)^2] + 10 \times x^3/3 + 313.33$

$= - 50 \times x^2 + 30x^2 - 120x + 120 + 10 \times x^3/3 + 313.33$

$0 = -20 \times x^2 - 120x + 3.33\, x^3 + 433.33$ — 01 Mark

Solving this equation by trial and error method.

$x = 2.89$ m

Hence deflection is maximum at distance 2.89 m from B

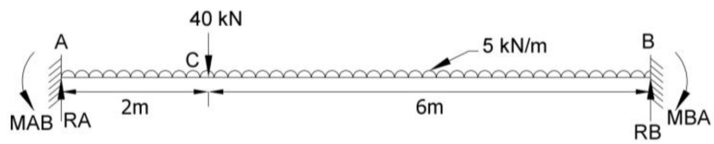
For y_{max}, put $x = 2.89$ in equ^n II

$y_{max} = 1/EI(-50 \times 2.89^3/3) + [10(2.89 - 2)^3] + 10 \times 2.89^4/12 + 313.33 \times 2.89$

$= 568.4 / EI$ — 01 Mark

Q.6 b) A fixed beam of span 8 m carries 5 kN/m udl over entire length along with a point load of 40 kN at 2m from left hand support. Find net BM at point load and draw BMD and SFD.

Ans



$M_{AB} = (40 \times 2 \times 6^2 / 8^2) + (5 \times 8^2 / 12)$ — 01 Mark

$= 71.67$ kN-m

$M_{BA} = (40 \times 2^2 \times 6 / 8^2) + (5 \times 8^2 / 12)$ — 01 Mark

$= 41.67$ kN-m

Reactions:

$\Sigma M_A = 0$

$40 \times 2 + 5 \times 8 \times 4 + 41.67 - 71.67 - R_B \times 8 = 0$

$R_B = (80 + 160 - 30) / 8$

$= 26.25$ kN. — 01 Mark

$R_A = 5 \times 8 + 40 - 26.25 = 53.75$ kN.

Bending moment at point load

$M_C = -71.67 + 53.75 \times 2 - 5 \times 2 \times 1$

$= 25.83$ kN-m — 01 Mark

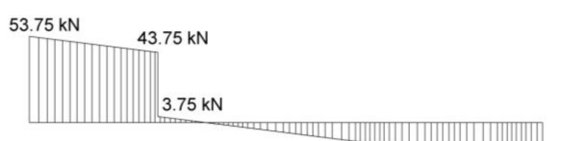
Shear force calculations:

At B = – 26.25 kN

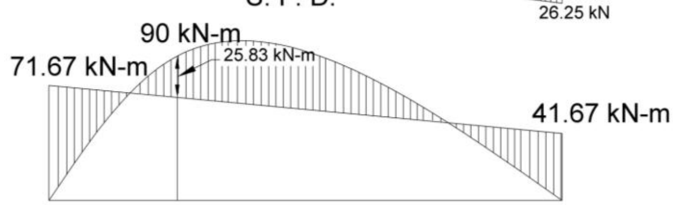
At C, just right = –26.25 + 5 x 6 = 3.75 kN — 02 Mark

At C, just left = –26.25 + 5 x 6 + 40 = 43.75 kN

At D = 53.75 kN



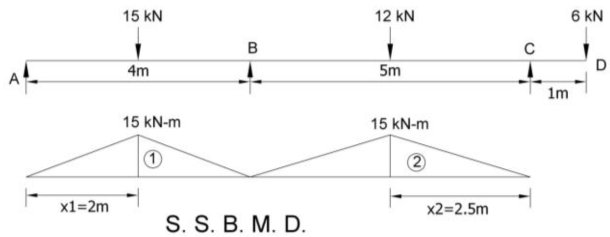
S. F. D. — 01 Mark



B. M. D. — 01 mark

Q.6 c) A beam ABCD is supported at A, B and C, CD being overhang AB = 4 m, BC = 5 m and CD = 1m AB and BC carries a central point of 15kN and 12kN respectively and a point load Of 6 kN at D. Calculate support moments using three moment theorem and draw SFD and BMD giving net BM.

Ans



S. S. B. M. D.

01 Mark

01 Mark

M_1 = 15 x 4/4 = 15.0 kN-m a_1 = 0.5 x 4 x 15 = 30.0 x_1 = 4/2 = 2.0 m

M_2 = 12 x 5/4 = 15.0 kN-m a_2 = 0.5 x 5 x 15 = 37.5 x_2 = 5/2 = 2.5 m   01 Mark

Using three moment theorem;

$M_A \times l_1 + 2M_B (l_1 + l_2) + M_C \times l_2 = -[(6 \times a_1 \times x_1/l_1) + 6 \times a_2 \times x_2/l_2)]$

M_A = (End simple supports)

$M_C = -6 \times 1 = -6.0$ kN-m

$2M_B(4 + 5) - 6 \times 5 = -[(6 \times 30.0 \times 2.0/4) + 6 \times 37.5 \times 2.5/5)]$

$18.0M_B = -90.0 - 112.5 + 30$ 01 Mark

$M_B = -172.5/18 = -9.583$ i.e. 9.583 kN-m Hogging

Reactions:

R_A = (15 x 2 – 9.583) / 4 = 5.1 kN.   01 Mark

R_C = (12 x 2.5 + 6 x 6 – 9.583) / 5 = 11.28 kN.

R_B = 15 + 12 + 6 – 11.28 – 5.1 = 16.62 kN.   01 Mark

Net bending moments:

Under 15 kN load = 15 – (9.583/2) = 10.21 kN-m

Under 12 kN load = 15 – [(9.583 + 6)/2] = 7.21 kN-m.

Shear force calculations:

At D = 6 kN

At C, right = 6.0 kN

At C, left = 6.0 -11.28 = – 5.28 kN   01 Mark

At 12 kN load, right = – 5.28 kN

At 12 kN load, left = – 5.28 + 12 = 6.72 kN

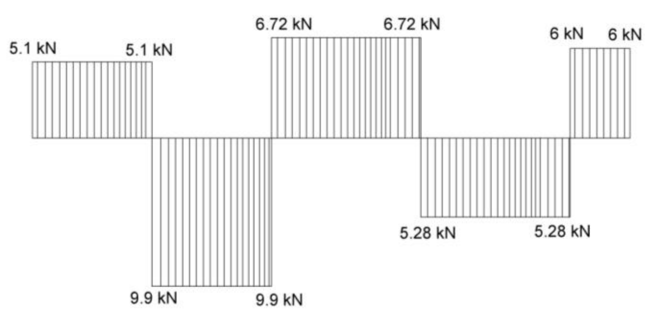
At B, right = 6.72 kN

At B, left = 6.72 – 16.62 = – 9.9 kN

At 15 kN load, right = – 9.9 kN

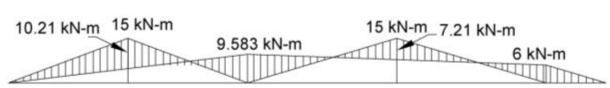
At 15 kN load, left = – 9.9 + 15 = 5.1 kN

At A = 5.1 kN



S . F. D.

01 Mark



B. M. D.

01 Mark