



SUMMER -2017 EXAMINATION

Model Answer

Page No: 01/33

Subject code: 17422 Subject : (Theory of Structures)

Important Instructions to examiners:

- Important Inst. actions to examiners:**

 - 1) The answer should be examined by keywords and not as word-to-word as given in the model answer scheme.
 - 2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
 - 3) The language error such as grammatical, spelling errors should not be given more importance. (Not applicable for subject English and communication skill).
 - 4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figure drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
 - 5) Credits may be given step wise for numerical problems. In some cases, the assumed constants values may vary and there may be some difference in the candidates answer and model answer.
 - 6) In case of some questions credit may be given by judgment on part of examiner of relevant answer based on candidates understanding

Question and sub Question	Model Answer/paper Solution with sketches	Marks
Q. 1 A	Attempt any Six of the following	12 Marks
a)	State the condition of stability of Masonry dam	
	Ans:	
	1. Stability against Overturning : $(P.h/3) \leq W(b-x)$	$\frac{1}{2} M$
	2. Stability against no Tension : $e \leq b$	$\frac{1}{2} M$
	3. Stability against Sliding : $P \leq F$	$\frac{1}{2} M$
	4. Stability against crushing	$\frac{1}{2} M$
b)	Define the slope and deflection of beam	1M
	Ans : Slope of a beam:-	
	Slope of a beam at a point is defined as rate of change of deflection with respect to longitudinal distance i.e.	
	slope dy/dx where, y = deflection The point of slope is " radian "	
	where π radian = 180 degrees	
	Deflection of a beam :-	

The vertical displacement of a point on a beam after loading with respect to its original position before loading is called deflection. It is denoted by 'y' unit is mm

- c) **What do you understand by boundary condition of a beam? State the boundary condition for two different nature of beam supports,**

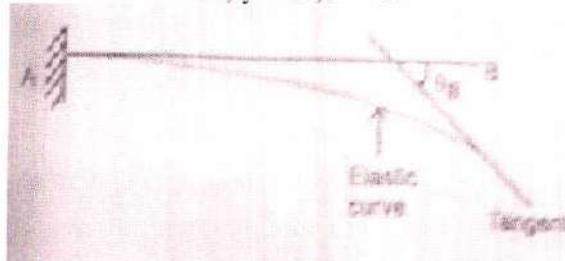
Ans : It is a general mathematical principle that the number of boundary condition necessary to determine a solution to a differential equation matches the order of the differential equation .

- Cantilever beam X is consider from free end

Boundary condition At $x = 0, y = y_{\max}, \theta = \theta_{\max}$

At $x = L, y = 0, \theta = 0$

1M

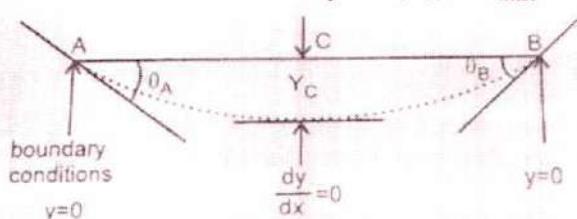


- Simply supported beam The condition to locate point of maximum deflection is slope of tangent at that point is zero

Boundary condition At $x = 0, y = 0, \theta_A = \theta_{\max}$

$x = L, y = 0, \theta_A = \theta_{\max}$

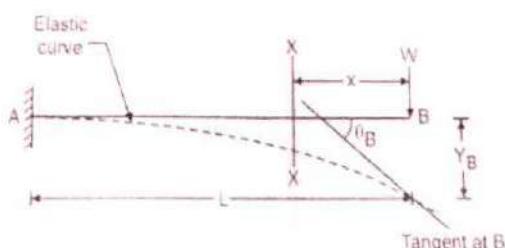
1M



- d) **State the values of maximum deflection for a cantilever beam of span L carrying a point load 'W' at a distance 'L' from the fixed end.**

Ans : The Values of maximum deflection for a cantilever beam of a span L carrying a point load 'W' at a distance 'L' from the fixed end

1M



$$Y_{max} = Y_B = - \frac{WL^3}{3EI}$$

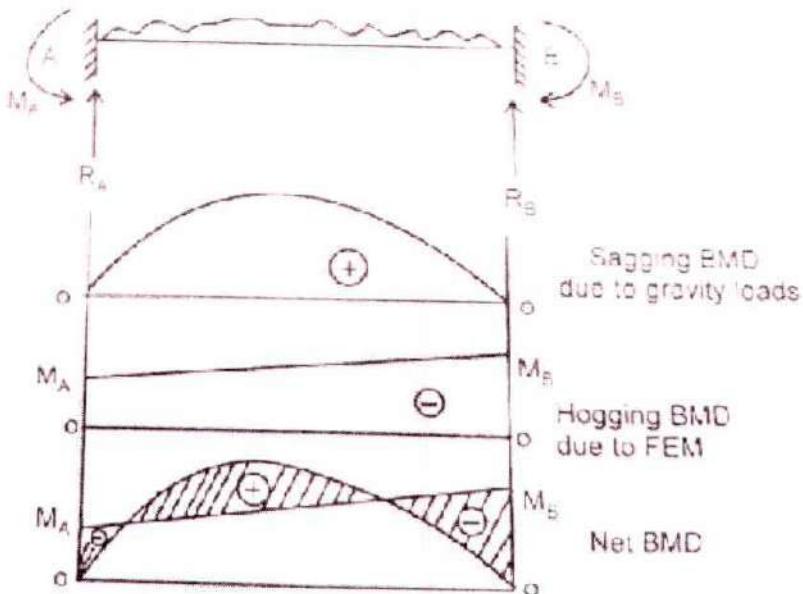
1M

Where , W = Point load in KN
L = Span in m
Y = Deflection

e) **State the Principle of Superposition**

Ans: Principle of Superposition:- It states that “ If number of forces are acting simultaneously on a body then their combined effect on the body is equal to the algebraic sum of the effects of the individual forces or moments considered separately

1M



1M

f) **Define i) Distribution Factor ii) Portal Frames :**

i) Distribution Factor :

It is the ratio of relative stiffness of a member to the total stiffness of all the members meeting at a joint

1M

ii) Portal Frames :

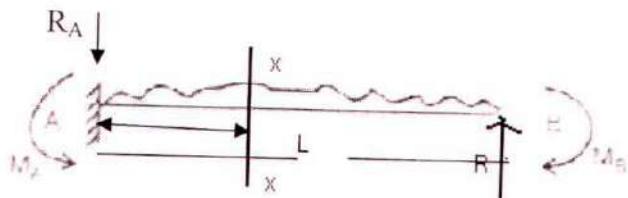
Portal frame is a structure of least two columns and a connecting beam .It is a two dimensional structure assumed separate from a structure for analysis

1M

g) **What is the carry over factor for a beam fixed at one end and simply supported at the other?**

Ans:

Consider any section x-x of x distance from A



1M

$$\frac{M_A}{M_B} = \frac{1}{2}$$

Where, M_A = Moments at A in KNm
 M_B = Moments at B in KNm

1M

h) Define perfect and imperfect frame

1) Perfect frame: A frame made up of just sufficient numbers so that it can remain in stable equilibrium, when loaded at joints.

The condition of frame to be stable is : $n = 2j - 3$

Where, n = numbers of members

j = number of joints

2) Imperfect frame : A frame made up of either more than or less than, just sufficient number of members to keep it in static equilibrium is called Imperfect frame . Where , $n < 2j - 3$ -----Deficient frame

1M

1M

$n > 2j - 3$ -----Redundant frame

Q.1 B) Attempt any Two of the following

8 M

a) Explain the points to be considered to design 30 storey building with respect to wind pressure.

.Ans : While designing a 30 storey building the points to be considered are :

1. As the building is greater than 10 m height .The wind pressure will be considered
2. Location of building and topography of area which decide the coefficient of wind pressure.
3. The wind pressure is assumed to vary uniformly with increase in pressure with respect to height of building.
4. The wind pressure will cause large bending stresses and the self weight of the building will cause direct stresses and the material should be capable of bearing the total maximum stresses at base.

4mark
(any 4
for each
1)

- b) A short column of external 250 mm and internal diameter 200 mm carries an eccentric load .Find the eccentricity which the load can have without producing tension in the section of a column

Given :-

$$D = 250 \text{ mm}$$

$$d = 200 \text{ mm}$$

$$\text{Solution: } -\sigma_{\min} = \sigma_d - \sigma_b$$

$$= P/A - \frac{Pe}{I} y$$

$$\frac{P}{\frac{\pi(D^2-d^2)}{4}} - \frac{Pe}{\frac{\pi(D^4-d^4)}{64}} \times D/2$$

----- (1)

2 M

For No Tension $\sigma_{\min} = 0$ put in eq (1)

$$e = \frac{(D^2-d^2)}{8D}$$

$$= \frac{(250^2-200^2)}{8 \times 25}$$

$$e = 51.25 \text{ mm}$$

2 M

- c) State four assumption made in the analysis of simple frame

Ans : Assumption made in analysis of frame are

1. The members are pin jointed at ends
2. The frame is perfect ,i.e. it satisfies $n = 2j-3$ condition
3. The load is acting at joints only.
4. Self weight of the member is neglected

4mark
(any 4
for each
1)

Q. 2 Attempt any Four of the following

- a) A chimney having diameter 5m and 50 m high . It is subjected to a horizontal wind pressure of 7KPa normal to the chimney ,Find the maximum bending stress in a chimney (use C = 0.70)

Ans : Given

$$D = 5 \text{ m}$$

$$H = 50 \text{ m}$$

$$\rho = 7 \text{ KPa} = 7 \times 10^3 \text{ N/m}^2$$

$$C = 0.7$$

Solution

$$\text{Maximum Bending stress , } \sigma_b = \frac{M}{I} \times y$$

$$1) \text{ Moment of 'P' about base } = P(H/2)$$

16 M

3) Total wind force $P = C \times \rho \times \text{Projected area}$

4) $I = \frac{\pi (D^4)}{64}$

$$= 0.7 \times 7000(5 \times 50)(50/2) \times 5/2$$

$$\frac{\pi (5^4)}{64}$$

$$\sigma_{\max} = 2.4950 \text{ N/m}^2$$

1 M

1 M

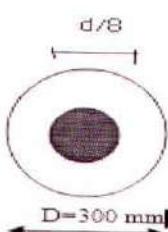
1 M

b) Calculate core of a section for following and draw neat sketches:

i. Circular section 300mm diameter

ii. Rectangular section is a 250 x 600 mm in size

Ans i) Core of section for a Circular section is a circle of diameter

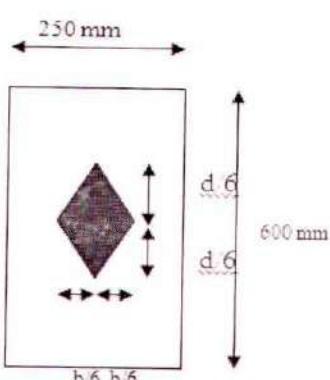


1 M

$$e_{\max} = \frac{D}{8} = \frac{300}{8} = 37.5 \text{ mm}$$

1 M

ii) Core of section for a rectangular section is a 250 x 600 mm



1 M

$$\text{i) } e_x = d/6 = 600/6 = 100 \text{ mm}$$

1 M

$$\text{ii) } e_y = \frac{b}{6} = \frac{250}{6} = 41.6667 \text{ mm}$$

- c) A square pillar 200 mm side is subjected to an eccentric load of 95 KN at an eccentricity of 50 mm in the plane bisecting thickness. Find the maximum and minimum intensities of stress at base section

Ans : Given

$$b = 200 \text{ mm}$$

$$d = 200 \text{ mm}$$

$$P = 95 \text{ KN}$$

$$e = 50$$

Ans :

Solution :

σ_0 = direct stress

σ_b = Bending stress

$$\text{i) } \sigma_{\max} = \text{direct stress} + \text{Bending stress}$$

$$= \sigma_0 + \sigma_b$$

$$= P/A + \frac{M}{I} y$$

$$= \frac{95 \times 10^3}{200 \times 200} + \frac{95 \times 10^3 \times 50}{200 \times 200^3} \times 200/2$$

$$= 2.375 + 3.56$$

$$= 5.93 \text{ N/mm}^2$$

$$\text{ii) } \sigma_{\min} = \text{direct stress} + \text{Bending stress}$$

$$= \sigma_0 - \sigma_b$$

$$= P/A - \frac{M}{I} y$$

$$= \frac{95 \times 10^3}{200 \times 200} - \frac{95 \times 10^3 \times 50}{200 \times 200^3} \times 200/2$$

$$= 2.375 - 3.56 \text{ (Comp)}$$

$$= -1.185 \text{ N/mm}^2 \text{ (Comp)}$$

- d) A cantilever of length 2m carries a UDL of 10 kn/m over a entire span .If the section is rectangle 100 mm wide and 300 mm deep . Find the slope at the free end. And deflection at free end Take $E = 100 \text{ KN/mm}^2$

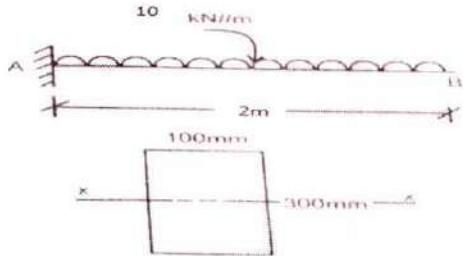
Given

$$E = 100 \text{ KN/mm}^2$$

$$= 1 \times 10^8 \text{ KN/m}^2$$

$$L = 2 \text{ m}$$

$$w = 10 \text{ kN/m}$$



Solution :

$$1) I = \frac{100 \times 300^3}{12}$$

$$= 225 \times 10^6 \text{ mm}^4$$

2) Slope at B,

$$\frac{dy}{dx} = - \frac{W L^3}{6 EI} = \frac{10 \times 2^3}{6 EI}$$

$$= - \frac{10 \times 2^3}{6 \times 1 \times 10^8 \times 2.25 \times 10^{-4}}$$

$$= - 5.94 \times 10^{-4}$$

$$= - 0.000592 \text{ radians}$$

3) Deflection at B

$$= - \frac{W L^4}{8 EI} = \frac{10 \times 2^4}{8 \times 1 \times 10^8 \times 2.25 \times 10^{-4}}$$

$$= - 8.88 \times 10^{-4} \text{ m}$$

$$= - 0.888 \text{ mm}$$

1M

1M

1M

1M

e)

A simply supported beam of 6m carries a UDL of 5 KN/m find the breadth and depth of beam , if maximum bending stress is not to exceed 8 N/mm² and maximum deflection 20 mm .Take E = 1×10^4 N/mm²

Given

$$\sigma_b = 8 \text{ N/mm}^2$$

$$= 8000 \text{ KN / m}^2$$

$$Y_{\max} = 20 \text{ mm}$$

$$E = 1 \times 10^4 \text{ N/mm}^2$$

$$= 1 \times 10^7 \text{ KN/m}^2$$

$$1) BM_{\max} = \frac{WL^2}{8} = \frac{5 \times 6^2}{8} = 22.5 \text{ KN/m}$$

$$M = 22.5 \text{ KN/m}$$

1M

$$2) I = \frac{bd^3}{12}$$

From Bending equation

1M

$$3) \sigma b = \frac{My}{I}$$

$$bd^2 = 0.0168 \quad \dots \dots \dots \quad (1)$$

From deflection equation

$$4) Y_{max} = \frac{-5WL^4}{384EI}$$

$$0.020 = \frac{5 \times 5 \times 6^4}{384 \times 1 \times 10^7 \times bd^3} \frac{12}{12}$$

1M

$$bd^3 = 5.06 \times 10^{-3} \quad \dots \dots \dots \quad (2)$$

Put equation (1) in equation (2)

$$0.0168 = 5.06 \times 10^{-3}$$

$$d = 0.301 \text{ m}$$

1M

$$d = 301 \text{ mm}$$

Put in equation (1)

$$b = 0.185 \text{ m}$$

$$b = 185 \text{ mm}$$

- f) Write Clapeyron's moment theorem for beam with different M.I. giving meaning of each term along with the diagram of beam
Ans ;

1M

Clapeyron's moment theorem for beam with different M.I.
Let ABC be a continuous beam, selected for analysis and loaded as shown below a₁ and a₂ are areas of sagging BMD

Here M_A and M_C = 0

The support are simple supports

Unknowns are R_A, R, R_C and M_B To obtain M_B Clapeyrons theorem used as below

$$\begin{aligned} M_A \frac{L_1}{I_1} + 2M_B \left(\frac{L_1}{I_1} + \frac{L_2}{I_2} \right) + M_C \frac{L_2}{I_2} \\ = -6 \left(\frac{a_1 x_1}{L_1 I_1} + \frac{a_2 x_2}{L_2 I_2} \right) \end{aligned}$$

Where, M_A = Support moment at A
M_A = Support moment at A

M_B = Support moment at B
 M_C = Support moment at C
 L_1 = Length of the span AB
 L_2 = Length of the span BC
 I_1 = Moment of Inertia for the span AB
 I_2 = Moment of Inertia for the span BC
 a_1 = area of moment diagram for the span AB
 x_1 = Centroidal distance of moment diagram for the span AB
 a_2 = area of moment diagram for the span BC
 x_2 = Centroidal distance of moment diagram for the span BC
 The term above used above are as shown in Figure,
 M_A and $M_C = 0$

1M

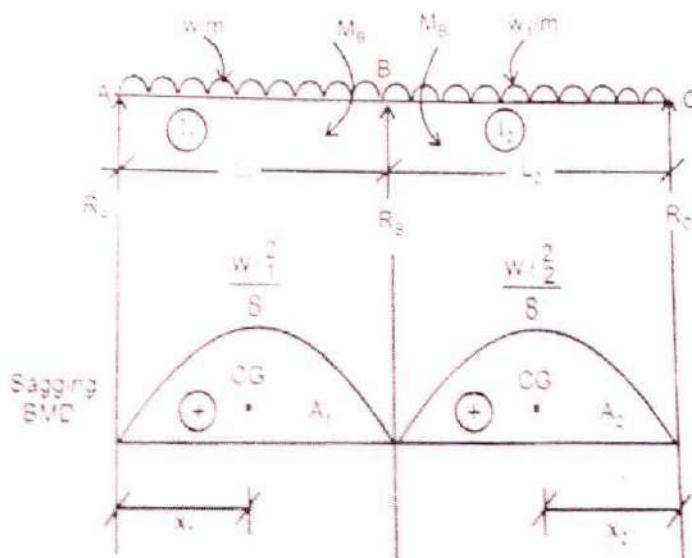
M_B can be obtained from the above equation. I_1 and I_2 are the moment of Inertia of beams respectively, If beams are of same size, then $I_1 = I_2 = I$ hence the equation reduces to

$$M_A \cdot L_1 + 2M_B (L_1 + L_2) + M_C \cdot L_2$$

1M

$$= -6 \left(\frac{a_1 x_1}{I_1} + \frac{a_2 x_2}{I_2} \right)$$

Thus for 'n' number of unknown moments , 'n' such equation are required



1M

Question	Description	Marks
Q.3 3(a)	<p>Attempt any FOUR</p> <p>A simply supported beam of span 6m, carries a point load of 60KN at 2m from left hand support. Calculate deflection under point load. Use Macaulay's method.</p> <p>To Find support reactions</p> $\sum MA = 0 \quad (\text{clockwise})$ $-RB \times 6 + 60 \times 2 = 0$ $[RB = 20\text{KN}]$ <p>To Find slope & deflection</p> $EI \frac{d^2y}{dx^2} = M \quad \text{- Differential equation.}$ $EI \frac{d^2y}{dx^2} = 40x - 60(x-2) \quad \text{- Moment eqn}$ <p>Integrating w.r.t. x</p> $EI \frac{dy}{dx} = 40x^2 + C_1 - 60(x-2)^2 + C_2 \quad \text{- Slope eqn. I}$ <p>Again Integrating w.r.t. x</p> $EI \cdot y = 20x^3 + C_1 x + C_2 - 30(x-2)^3 + C_3 \quad \text{- Deflection eqn. II}$ <p>To calculate Constants of Integration</p> <p>Boundary Conditions:</p> <p>At $x=0, y=0$ Putting in Deflection eqn. I</p> $EI(0) = \frac{20(0)^3}{3} + C_1(0) + C_2 - \frac{30}{3}(0)^2 = 0$ $C_2 = 0$	16 Marks
1 Mark		

At $x=6m$, $y=0$ putting in deflection eqⁿ

$$EI(0) = \frac{20(6)^3}{3} + C_1(06) + C_2 \left| -\frac{30}{3}(6-2)^3 \right.$$

$$0 = 800 + 6C_1$$

$$\boxed{C_1 = -133.33}$$

1 mark

Putting values of C_1 and C_2 in slope and deflection equation & rewriting eqⁿ

$$EI \cdot \frac{dy}{dx} = 20x^2 - 133.33 \left| -30(x-2)^2 \right| \begin{matrix} \textcircled{1} \\ \textcircled{2} \end{matrix} \begin{matrix} \text{Final slope eq}^n \\ \text{eq}^n \end{matrix}$$

$$EI \cdot y = \frac{20}{3}x^3 - 133.33x + 0 \left| -\frac{30}{3}(x-2)^3 \right. \begin{matrix} \textcircled{1} \\ \textcircled{2} \end{matrix} \begin{matrix} \text{final defl. eq}^n \\ \text{eq}^n \end{matrix}$$

4) Deflection under point load

At $x=2m$, $y=y_c$ putting in final deflection eqⁿ

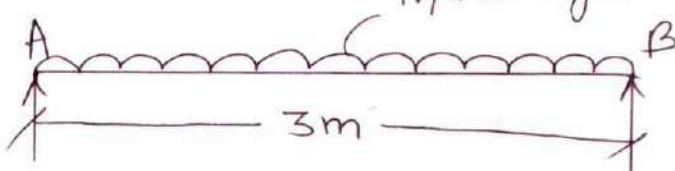
$$EI \cdot y_c = \frac{20}{3}(2)^3 - 133.33(2) - 0$$

$$\boxed{y_c = -\frac{213.326}{EI}}$$

— Deflection below point load, 1 mark

Q. 3(b)

A simply supported beam having 3m span carries a UDL 'W' per unit length. If slope at the end is not to exceed 1.5° . Find the maximum deflection.
w/unit length



$$\text{slope } (0) = 1.5^\circ$$

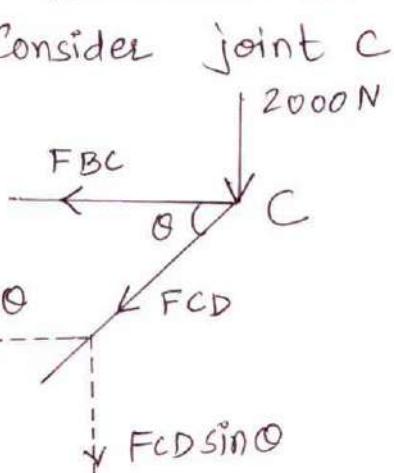
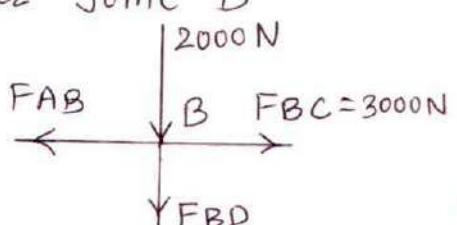
$$\theta = 1.5 \times \frac{\pi}{180} \text{ radians}$$

$$\theta = 0.02618 \text{ radians.}$$

$\frac{1}{2}$ mark

Question	Description	Marks
	slope of the beam is calculated as $\theta = \frac{WL^3}{24EI}$ $0.02618 = \frac{W \times 3^3}{24 EI}$ $\therefore \left[\frac{W}{EI} = 0.02327 \right]$	$\frac{1}{2}$ Mark
	Maximum Deflection of beam $y_{max} = \frac{5WL^4}{384EI}$ $y_{max} = \frac{5 \times W (3)^4}{384EI}$	1 Mark
	Putting value of $\frac{W}{EI}$ in deflection equation $y_{max} = \frac{5 \times 0.02327 \times (3)^4}{384}$ $y_{max} = 0.02454 \text{ m}$ $\therefore \left[y_{max} = 24.54 \text{ mm} \right]$	1 Mark
Q.3(c)	State two advantages and two disadvantages of fixed beam over simply supported beam. <u>Advantages of fixed Beam.</u> 1) The slopes at the end support in case of fixed beam are zero but in case of simply supported beam slopes are maximum. 2) The maximum bending moment and deflection are less as compared to simply supported beam for same span and loading. 3) Fixed beam is more strong, stable and stiff than simply supported beam.	Any Two for two marks.

Question	Description	Marks
	4) The cross section required for fixed beam is smaller and steel required is less due to lesser B.M. and hence it is economical as compared to simply supported beam.	
	<u>Disadvantages of fixed beam</u> -	
	1) If any one of the support sinks to a small extent, it induces additional moment at each end.	Any TWO for Two marks.
	2) Since both the end of beam are fixed, temperature stresses are developed due to variation in temperature.	
	3) Complete fixity cannot be achieved.	
Q.3(d)	A fixed beam of span 8m carries a point load 'W' at a distance 'x' from the left hand support. If the moment at the left end is twice that at the right end evaluate 'x'	
	$MA = 2MB \quad \text{--- (I)}$	1 Mark
	To find fixed end moments.	
	$MA = -\frac{Wab^2}{L^2} = -\frac{W(x)(8-x)^2}{8^2}$	$\frac{1}{2}$ Mark
	$MB = -\frac{Wa^2b}{L^2} = -\frac{W(x^2)(8-x)}{8^2}$	$\frac{1}{2}$ Mark
	Putting values of MA & MB in equation I	
	$\therefore -MA = 2MB$	
	$\frac{W(x)(8-x)^2}{8^2} = 2 \left[\frac{W(x^2)(8-x)}{8^2} \right]$	1 Mark

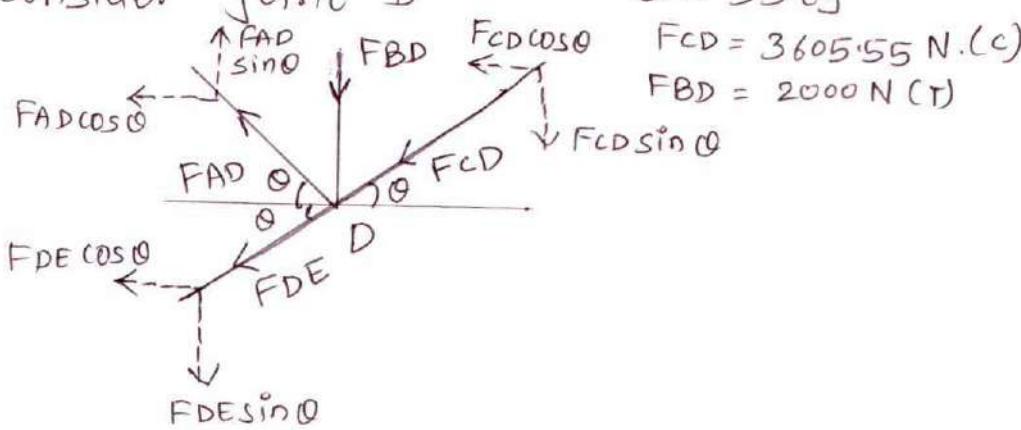
Question	Description	Marks
	$(8-x) = 2x$ $3x = 8$ $x = \frac{8}{3}$ $x = 2.67\text{ m}$	1 Mark
Q.3(e)	Using method of joints, find nature and magnitude of forces in member AB and member DE of frame.	
	Slope (θ) = $\tan^{-1}(\frac{4}{6})$ $\theta = 33.69^\circ$ <p>1) Consider joint C</p>  $\sum F_x = 0$ $-F_{BC} - F_{CD} \cos \theta = 0 \quad \text{(I)}$ $\sum F_y = 0$ $-2000 - F_{CD} \sin \theta = 0$ $F_{CD} = \frac{-2000}{\sin 33.69}$ $[F_{CD} = -3605.55 \text{ N}] \text{ (Compression)}$ Putting value of F_{CD} in eq: (I) $-F_{BC} + 3605.55 \cos 33.69 = 0$ $[F_{BC} = 3000 \text{ N}] \text{ (Tension)}$ 2) Consider Joint B  $\sum F_x = 0$ $+3000 - F_{AB} = 0$ $[F_{AB} = 3000 \text{ N}] \text{ (Tension)}$ $\sum F_y = 0$ $-F_{BD} - 2000 = 0$ $[F_{BD} = -2000 \text{ N}] \text{ (Compression)}$	1 Mark

Question

Description

Marks

Consider joint D



$$\theta = 33.69^\circ$$

$$F_{CD} = 3605.55 \text{ N. (c)}$$

$$F_{BD} = 2000 \text{ N (T)}$$

$$\sum F_x = 0$$

$$-F_{AD} \cos \theta - F_{CD} \cos \theta - F_{DE} \cos \theta = 0$$

$$-F_{AD} - F_{CD} - F_{DE} = 0$$

$$F_{AD} + F_{DE} = -3605.55 \quad \text{--- (I)}$$

$$\sum F_y = 0$$

$$F_{BD} - 3605.55 \sin \theta - F_{DE} \sin \theta + F_{AD} \sin \theta = 0$$

$$-2000 - 3605.55 \sin 33.69^\circ - F_{DE} \sin 33.69^\circ + F_{AD} \sin 33.69^\circ$$

$$-2000 - 2000 - F_{DE} \sin 33.69^\circ + F_{AD} \sin 33.69^\circ = 0$$

$$\therefore F_{AD} - F_{DE} = 7211.11 \quad \text{--- (II)}$$

Addition of eqn (I) & (II)

$$F_{AD} + F_{DE} = -3605.55$$

$$F_{AD} - F_{DE} = 7211.11$$

$$2F_{AD} = +3605.55$$

$$[F_{AD} = 1802.78 \text{ N}] - (\text{Tension})$$

Putting value of FAD in eqn (I)

$$1802.78 + F_{DE} = -3605.55$$

$$[F_{DE} = -5408.33 \text{ N}] \text{ Compression.}$$

1 Mark

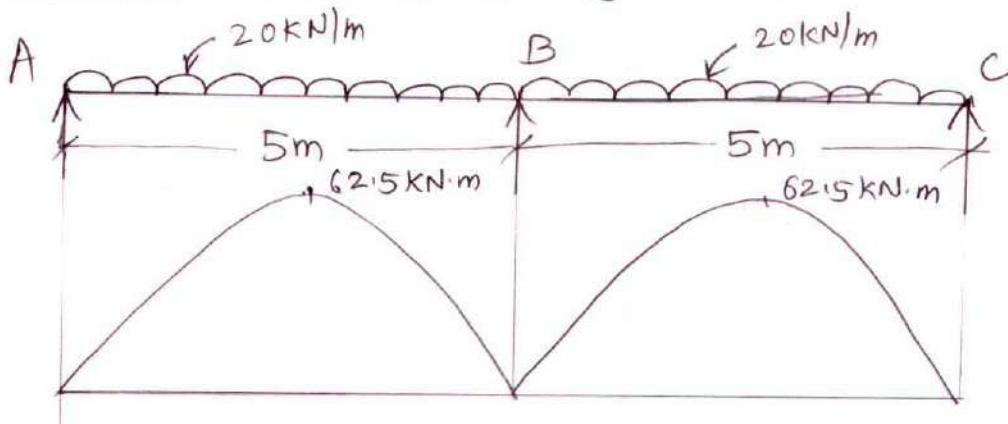
MEMBER	FORCE	NATURE
AB	3000 N	Tension.
DE	5408.33 N	Compression.

Question	Description	Marks
Q.3(f)	Using Method of section calculate the magnitude of forces and its nature of member BC, BE & AE	
	<p>RA = 2.5 kN.</p> <p>To calculate support reactions. 5</p> $RA + RB = 2 + 4 = 0.6$ <p>Taking moment at A</p> $\sum MA = 0 \quad (\text{F} \rightarrow)$ $-RD \times 6 + 4 \times 4.5 + 2 \times 1.5 = 0$ $[RD = 3.5 \text{ kN}]$ $\therefore RA = 0.6 - 3.5 =$ $[RA = 2.5 \text{ kN}]$	1 Mark
	<p>Consider section ①-① passing through member BC, BE & AE assuming all forces tensile.</p> <p>Taking moment At B considering left side of section.</p> $\sum MB = 0 \quad (\text{F} \rightarrow)$ $2.5 \times 1.5 - FAE \times h_1 = 0$ $3.75 - 2.598 FAE = 0$ $\therefore [FAE = +1.443 \text{ kN}] \quad (\text{Tension})$ $\sum ME = 0$ $2.5 \times 3 - 2 \times 1.5 + FBC \times h_2 = 0$ $\sin 60^\circ = \frac{h_1}{3}$ $h_1 = 2.598 \text{ m}$	1 Mark
	$\sin 60^\circ = \frac{h_2}{3}$ $h_2 = 2.598 \text{ m}$	

Question	Description	Marks												
	$7.5 - 3 + 2.598 F_{BC} = 0$ $F_{BC} = -1.732 \text{ kN}$ — compression. 1 Mark $\sum MA = 0$ $2 \times 1.5 - 1.5 \times 3 + F_{BE} \times h_3 = 0$ $\sin 60 = \frac{h_3}{3}$ $-0.75 + 2.598 F_{BE} = 0$ $[h_3 = 2.598 \text{ m}]$ $F_{BE} = 0.577 \text{ kN}$ — Tension. 1 Mark <table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th>Member</th> <th>Force</th> <th>Nature</th> </tr> </thead> <tbody> <tr> <td>AE</td> <td>1.443 kN</td> <td>Tension</td> </tr> <tr> <td>BC</td> <td>1.732 kN</td> <td>Compression</td> </tr> <tr> <td>BE</td> <td>0.577 kN</td> <td>Tension.</td> </tr> </tbody> </table>	Member	Force	Nature	AE	1.443 kN	Tension	BC	1.732 kN	Compression	BE	0.577 kN	Tension.	
Member	Force	Nature												
AE	1.443 kN	Tension												
BC	1.732 kN	Compression												
BE	0.577 kN	Tension.												

Q. 4 Attempt Any FOUR 16 Marks

4(a) A continuous beam ABC is supported on three supports at same level AB = BC = 5m Both spans carry UDL of intensity 20 kN/m over entire span. Calculate moment at B using theorem of three moments.

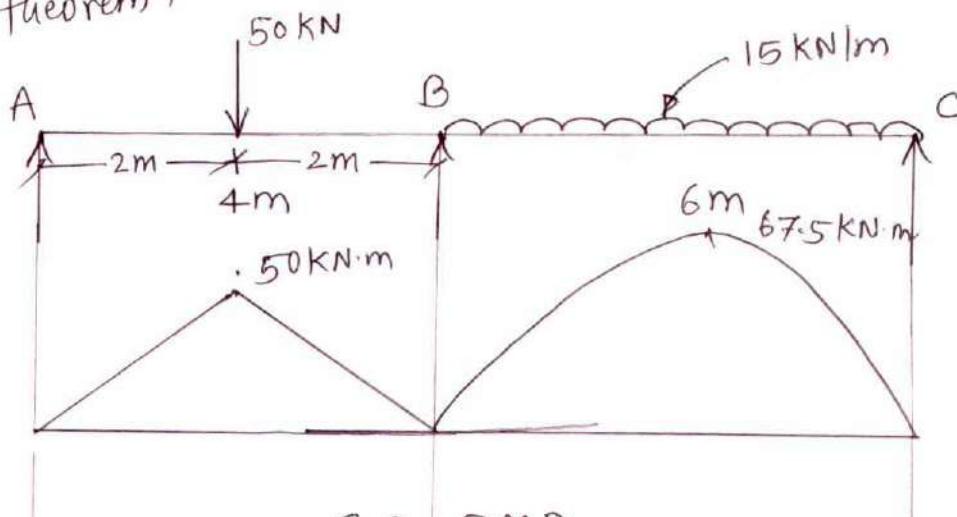


- 1) To Calculate S.S. Bending moments for Span AB & BC

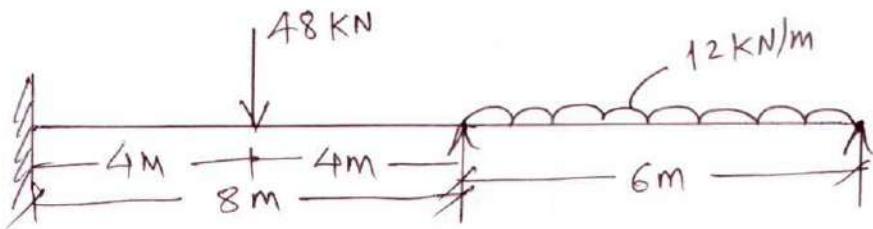
$$M_{max} = \frac{WL^2}{8} = \frac{20 \times 5^2}{8} = 62.5 \text{ kN.m}$$

1 Mark

Question	Description	Marks.
	Applying Three moment theorem for span AB & BC	
	$M_A \frac{1}{3}(L_1) + 2M_B(L_1 + L_2) + M_C \frac{1}{3}(L_2) = -\left[\frac{6A_1 \bar{x}_1}{L_1} + \frac{6A_2 \bar{x}_2}{L_2} \right]$	1 Mark
	$M_A = M_C = 0$ for simple supports.	
	$A_1 = \frac{2}{3} b h = \frac{2}{3} \times 5 \times 62.5 = 208.33 \text{ KN}\cdot\text{m}^2$	
	$\therefore A_2 = 208.33 \text{ KN}\cdot\text{m}^2$	
	$\bar{x}_1 = \bar{x}_2 = \frac{5}{2} = 2.5 \text{ m}$	
	$\therefore 2M_B(5+5) = -\left[\frac{6 \times 208.33 \times 2.5}{5} + \frac{6 \times 208.33 \times 2.5}{5} \right]$	1 Mark
	$20M_B = -1249.98$	
	$M_B = 62.5 \text{ KN}\cdot\text{m}$ — Support moment.	1 Mark
Q.4(b)	Draw deflected shape of two span continuous beam ABC, fixed at 'A' and hinged at 'C' Continuous over B. The beam AB is subjected to UDL over entire span.	
	<p>Diagram showing a two-span continuous beam ABC. Span AB is fixed at A and hinged at C. Span BC is hinged at B and fixed at C. A Uniformly Distributed Load (UDL) acts downwards over the entire length of span AB. The beam has a parabolic deflected shape. Slopes are labeled θ_A at A, θ_{B1} at B, and θ_C at C. The horizontal distance between supports is divided into L_1 and L_2.</p>	
	for sketch of beam —	2 marks
	for deflected shape and slopes —	2 marks

Question	Description	Marks
Q.4(c)	<p>A beam ABC is simply supported at A, B & C. Span AB and BC are of lengths 4m and 6m respectively. AB carries a central point load of 50 kN and BC carries a UDL of 15 kN/m over the entire span. Calculate support moment at B using three moment theorem.</p>  <p>S.S. BMD.</p> <p>1) Calculate S.S. Bending moments.</p> <p>for span AB</p> $M_{max} = \frac{WL}{4} = \frac{50 \times 4}{4} = 50 \text{ kN.m}$ <p>for span BC</p> $M_{max} = \frac{WL^2}{8} = \frac{15 \times 6^2}{8} = 67.5 \text{ kN.m}$ <p>2) Applying Three moment theorem for span AB & BC</p> $M_A \times L_1 + 2M_B(L_1 + L_2) + M_C(L_2) = -\left[\frac{6A_1\bar{x}_1}{L_1} + \frac{6A_2\bar{x}_2}{L_2} \right]$ <p>$M_A = M_C = 0$ - simple supports.</p> $A_1 = \frac{1}{2} \times 4 \times 50 = 100 \text{ kN.m}^2$ $A_2 = \frac{2}{3} b h = \frac{2}{3} \times 6 \times 67.5 = 270 \text{ kN.m}^2$ $\bar{x}_1 = \frac{4}{2} = 2 \text{ m} \quad \bar{x}_2 = \frac{6}{2} = 3 \text{ m}$	<p>1 Mark</p> <p>1 Mark</p> <p>1 Mark</p>

Question	Description	Marks.
	$O \times (L_1) + 2 M_B (6+4) + M_O (6) = - \left[\frac{6 \times 100 \times 2}{4} + \frac{6 \times 270 \times 3}{6} \right]$ $20 M_B = [300 + 810]$ $[M_B = 55.5 \text{ kN}\cdot\text{m}]$ — Support moment.	
Q.4(d)	Define carryover factor and stiffness factor with respect to moment distribution Method.	1 Mark.
	<u>Carryover Factor</u> := The ratio of moment produced at a joint to the moment applied at the other joint, without displacing it. is called as Carryover factor.	2 Marks
	<u>Stiffness Factor</u> := It is the moment required to rotate the end, while acting on it, through a unit angle without translation of the end. OR The moment required at the simply supported end of a beam so as to produce unit rotation at that end without translation of the either end is called stiffness factor	2 Marks.
Q.4(e)	A Continuous beam of uniform flexural rigidity is fixed at A and supported over B & C such that AB = 8m, BC = 6m. A UDL of 12kN/m acts on AB and a point load of 48kN acts at the centre of BC. Calculate distribution factor using MDM.	



Joint	Member	Stiffness Factor	Total Stiffness	Distribution Factors	
B	BA	$K_{BA} = \frac{4EI}{L} = \frac{4EI}{8}$ $K_{BA} = 0.5EI$	$\sum K_B = K_{BA} + K_{BC}$	$DF_{BA} = \frac{K_{BA}}{\sum K_B}$ $DF_{BA} = \frac{0.5EI}{1.0EI}$ $DF_{BA} = 0.5$	2 Mark
	BC	$K_{BC} = \frac{3EI}{L} = \frac{3EI}{6}$ $K_{BC} = 0.5EI$	$\sum K_B = 0.5EI + 0.5EI$ $\sum K_B = 1.0EI$	$DF_{BC} = \frac{K_{BC}}{\sum K_B}$ $DF_{BC} = \frac{0.5EI}{1.0EI}$ $= 0.5$	2 Mark
$\therefore DF_{BA} = 0.50$					
$DF_{BC} = 0.50$					

Q. 4 (f)

Explain the procedure of Moment Distribution Method.

Procedure -

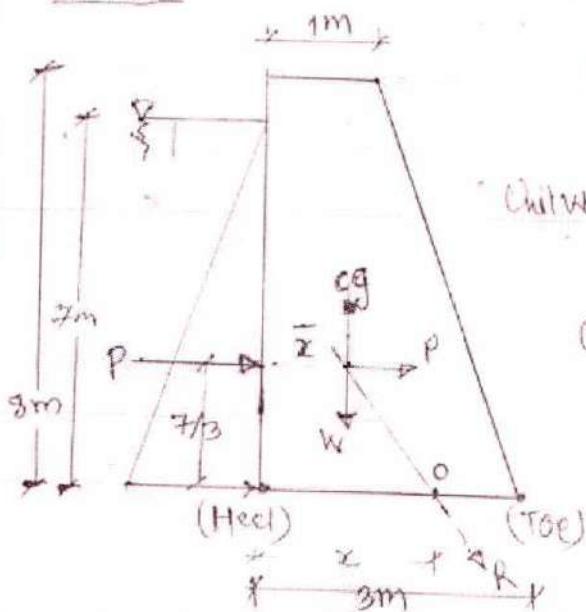
- 1) Assume all the supports are to be fixed and find fixed end moments (FEM) acting at the joints by using Hardy Cross sign convention. 1/2 Mark
- 2) Calculate the stiffness factors (K) and distribution factors (DF) for all members by considering the intermediate support as a joint. 1/2 Mark
- 3) Prepare moment distribution table consisting of Distribution Factors, fixed end moments. 1 Mark
- 4) Release the joints by applying an equal & opposite moment in case of simply support. Take carryovers to the opposite joints for fixed end & continuous end. 1 Mark
- 5) The method of balancing at the joints & carryovers to opposite joints should be done till it gives the required degree of accuracy of final moments. 1 Mark
The final moments are obtained by the algebraic sum of end moments of all members mentioned in table.

Q.5

Attempt Any TWO

Q.5a)

Given

Unit wt. of water = 10 kN/m^3 Unit wt. of masonry = 22 kN/m^3 Considering length of
dam = 1m

SOLN

1) Self wt. of Dam :-

$$W = 22 \left[\frac{1.5 + 3}{2} \times 8 \right] 1 = \underline{\underline{396 \text{ kN}}} \quad (1 \text{ M})$$

Acting at CG of section,

$$\bar{x} = \frac{(1.5)^2 + (1.5 \times 3) + (3)^2}{3 [1.5 + 3]} = \underline{\underline{1.17 \text{ m}}} \quad (1 \text{ M})$$

2) Force due to water

$$P = (10 \times 7) \frac{7}{2} = \underline{\underline{245 \text{ kN}}} \quad \text{acts } \frac{7}{2} \text{ from base} \quad (1 \text{ M})$$

of dam.

3) Point where resultant cuts the base,-

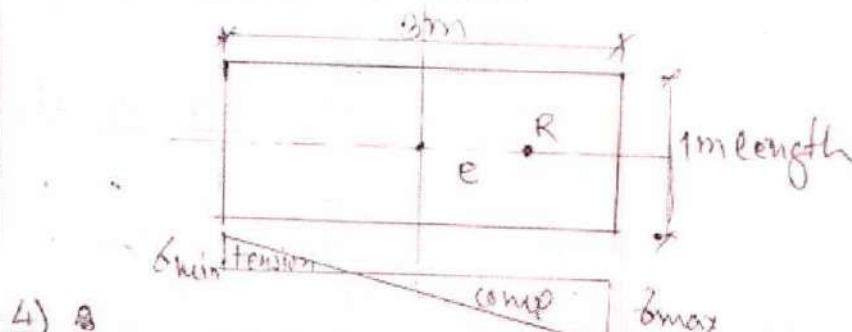
Taking moment @ 'O'

$$\Sigma M @ O = 0 = +(245 \times \frac{7}{3}) - (396 (x - 1.17)) \quad (1 \text{ M})$$

$$\therefore x = 2.61 \text{ m}$$

$$\therefore \text{Eccentricity } e = x - b/2 = 2.61 - \frac{3}{2} = \underline{\underline{1.11 \text{ m}}} \quad (1 \text{ M})$$

Dam section at base



(1M)

4) a

stresses develops at base

At toe side

$$\delta_{\max} = \frac{396}{3} \left[1 + \frac{6(1.11)}{3} \right]$$

$$\therefore \underline{\delta_{\max} = +425.40 \text{ kN/m}^2 \text{ (comp)}}$$

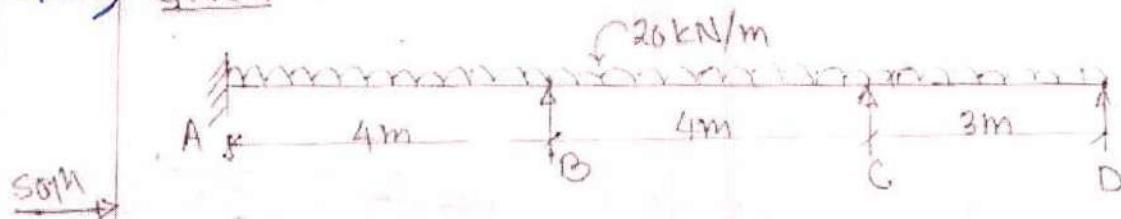
(9M)

& At heel side

$$\delta_{\min} = \frac{396}{3} \left[1 - \frac{6(1.11)}{3} \right]$$

$$\therefore \underline{\delta_{\min} = -161.40 \text{ kN/m}^2 \text{ (Tension)}}$$

(1M)

Q5b) Given

1) Calculation of Distribution Factor (DF)

Joint	Members	Relative Stiffness	Total Stiff.	DF.
B	BA	$\frac{4EI}{4} = EI$	2EI	0.5
	BC	$\frac{4EI}{4} = EI$		0.5
C	CB	$\frac{4EI}{4} = EI$	2EI	0.5
	CD	$\frac{3EI}{3} = EI$		0.5

(1/2 M)

(1/2 M)

(1/2 M)

(1/2 M)

2) Calculation of fixed end moments (FEM)

$$M_{AB} = \frac{-20 \times (4)^2}{12} = -26.67 \text{ kN}\cdot\text{m} \text{ (Anticlockwise)}$$

(1/2 M)

$$M_{BA} = +\frac{20 \times (4)^2}{12} = +26.67 \text{ kN}\cdot\text{m} \text{ (Clockwise)}$$

$$M_{BC} = \frac{-20 \times (4)^2}{12} = -26.67 \text{ kN}\cdot\text{m} \text{ (Anticlockwise)}$$

(1/2 M)

$$M_{CB} = +\frac{20 \times (4)^2}{12} = +26.67 \text{ kN}\cdot\text{m} \text{ (Clockwise)}$$

(1/2 M)

$$M_{CD} = \frac{-20 \times (3)^2}{12} = +15 \text{ kN}\cdot\text{m} \text{ (Anticlockwise)}$$

$$M_{DC} = +\frac{20 \times (3)^2}{12} = +15 \text{ kN}\cdot\text{m} \text{ (Clockwise)}$$

(1/2 M)

3) Calculation of sagging BM

$$M_{AB}^+ = M_{BC}^+ = \frac{20 \times (4)^2}{8} = 40 \text{ kN}\cdot\text{m}$$

(1/2 M)

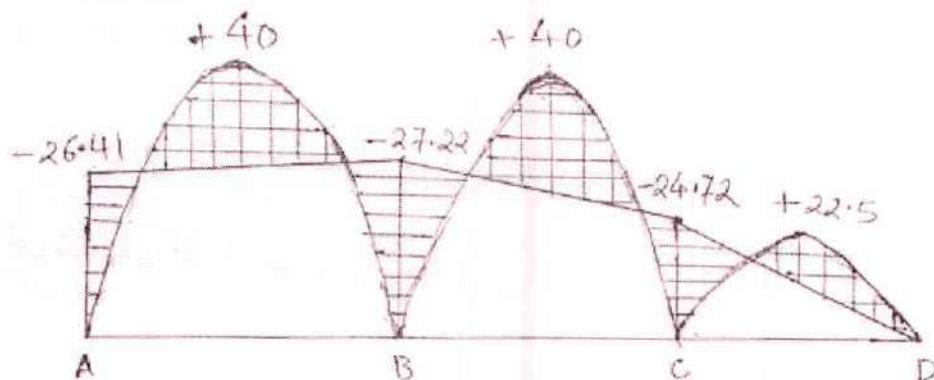
$$2) M_{CB}^+ = \frac{20 \times (3)^2}{8} = 22.5 \text{ kN}\cdot\text{m}$$

(1/2 M)

4) Calculation of support moment by MDM.

Joint	A	B	C	D	
D.F.	0	0.5 ↑	0.5	0.5 0.5	0
FEM	-26.67	+26.67	-26.67	+26.67 +15	+15
Released D				-7.5 ↙	-15 (1/2M)
Initial Mom.	-26.67	+26.67	-26.67	+26.67 -22.5	0
Balance				-2.085	-2.085
CO		-1.643 ↗			
Balance		+0.521	+0.521 ↘		
CO	0.261 ↙			+0.261	
Balance				-0.130 -0.13	(1/2M)
CO		-0.065 ↗			
Balance		+0.033	+0.033		
Final MDM	-26.41	+27.22	+27.22	+24.72 -24.72	0 (1M)

5)

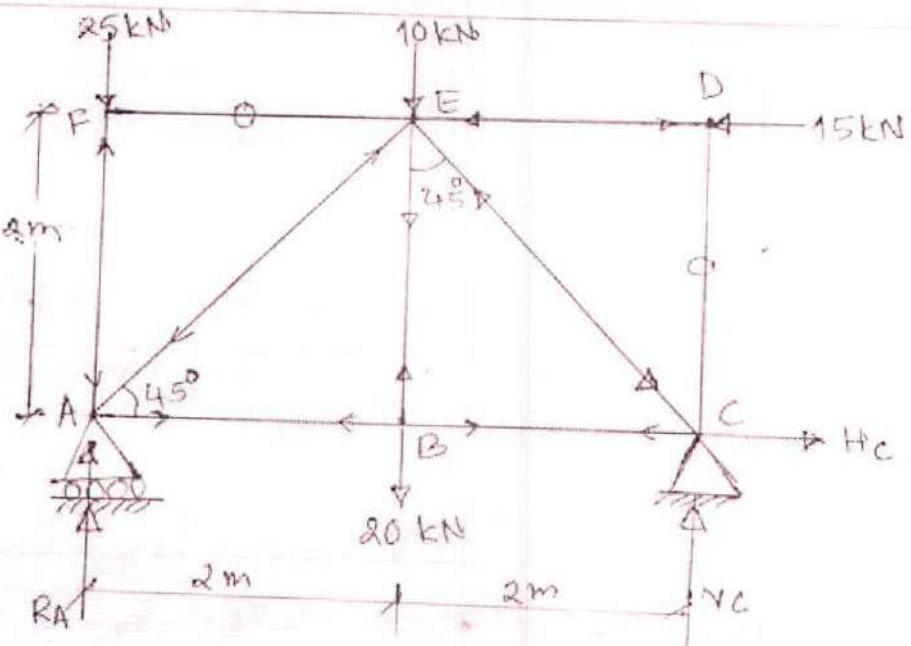


(1M)

BMD (KN·m) + sagging
- hogging

Q.5C)

Given



(3 M)

- 1) Using Equations of equilibrium, support reactions of frame, -

Taking moment from pt A

$$\Sigma M_{AA} = 0 = 25 \times 0 + (10 + 20) \times 2 - (15 \times 2) - (V_C \times 4)$$

$$\therefore V_C = 7.5 \text{ kN}$$

(3 M)

$$\Sigma F_x = 0 = +H_C - 15 = 0$$

$$\therefore H_C = +15 \text{ kN}$$

(3 M)

$$\& \Sigma F_y = 0 = R_A + V_C - 25 - 10 - 20 = 0$$

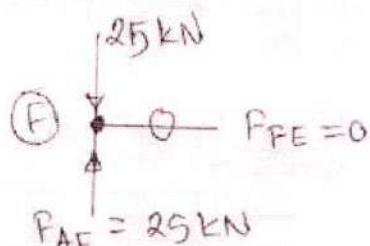
$$\therefore R_A = 47.5 \text{ kN}$$

(3 M)

- 2) Using Method of Joint forces in Frame.

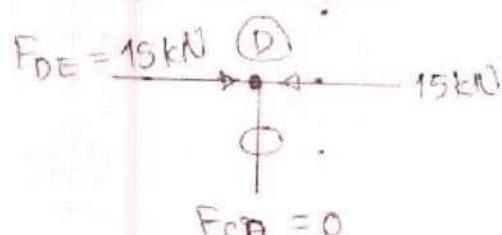
As all joints are in equilibrium using conditions of equilibrium at every joint

i) At jt F



$$F_{AF} = 25 \text{ kN}$$

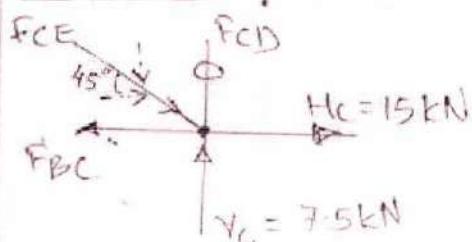
ii) At jt D



$$F_{CD} = 0$$

(1 M)

(iii) At Jl C



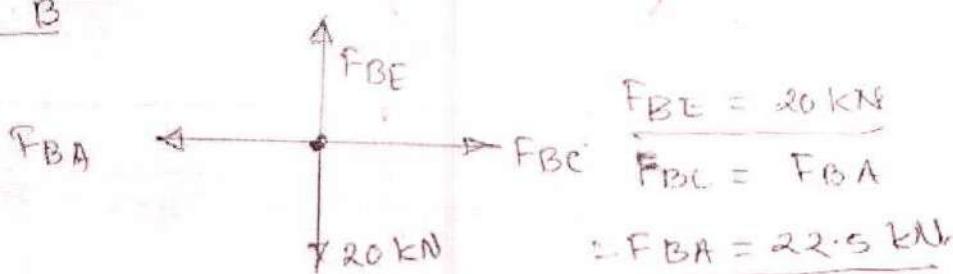
$$F_{CE} \sin 45^\circ = 7.5$$

$$\underline{F_{CE} = 10.61 \text{ kN}} \quad (1\text{M})$$

$$V_c = 7.5 \text{ kN} \quad \therefore F_{BC} = 15 + F_{CE} \cos 45^\circ$$

$$\therefore \underline{F_{BC} = 22.5 \text{ kN}}$$

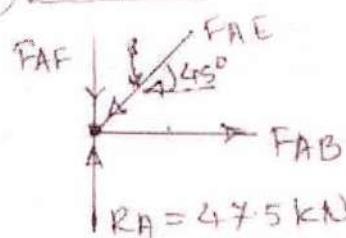
(iv) At Jl B



$$\frac{F_{BE} = 20 \text{ kN}}{F_{BC} = F_{BA}}$$

$$\therefore \underline{F_{BA} = 22.5 \text{ kN}}$$

(v) At Jl A

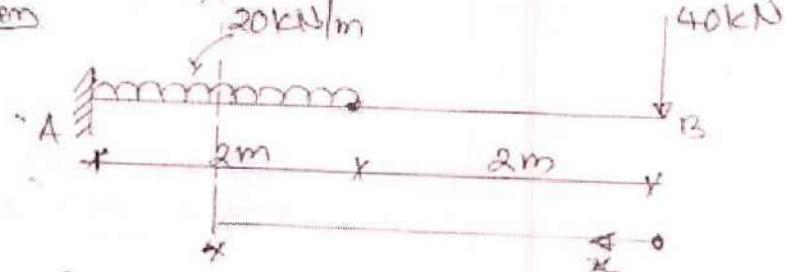


$$F_{AE} \cos 45^\circ = 22.5$$

$$\therefore \underline{F_{AE} = 31.82 \text{ kN}} \quad (1\text{M})$$

Member	Force (kN)	Nature
AB	22.5	Tension
BC	22.5	Tension
CD	0	-
DE	15	Compression
FF	0	-
AF	25	Compression
AE	31.82	Compression
BE	20	Tension
CE	10.61	Compression

(2M)

Q.6Attempt Any TWO**Q.6 a)**GivenSoln

Take free end as origin and consider a section xx at a distance ' x ' from free end as shown in fig.

$$EI \left(\frac{d^2y}{dx^2} \right) = Mac$$

$$= -40x + 20 \frac{(x-2)^2}{2} \quad (1\text{M})$$

Integrating w.r.t. x ,

$$EI \left(\frac{dy}{dx} \right) = -40 \frac{x^2}{2} + C_1 = -20x^2 + C_1 \quad \dots \quad (1\text{M})$$

$$\therefore EI \left(\frac{dy}{dx} \right) = -20x^2 + C_1 - 3.33(x-2)^3 \quad \dots \quad (1\text{M})$$

Integrating w.r.t. x ,

$$\therefore EI(y) = -20 \frac{x^3}{3} + C_1 x + C_2 - 3.33 \frac{(x-2)^4}{4} \quad (1\text{M})$$

$$\therefore EI(y) = -6.67x^3 + C_1 x + C_2 - 0.833(x-2)^4 \quad \dots \quad (1\text{M})$$

Where C_1 & C_2 are integration constants.

Using Boundary Conditions find C_1 & C_2 ,

At fixed end $\frac{dy}{dx} = 0$ at $x = 4\text{m}$.

$$\therefore EI(0) = -20(4)^2 + C_1 - 3.33(4-2)^3 \quad (1\text{M})$$

$$\therefore C_1 = +346.64 \quad (1\text{M})$$

$$4 \quad y=0 \quad \text{at} \quad x=4\text{m}$$

$$\therefore EI(0) = -6.67(4)^3 + (346.64 \times 4) + C_2 - 0.833 \frac{(4-2)^4}{4} \quad (1\text{M})$$

$$\therefore C_2 = -946.56 \quad (1\text{M})$$

Max. slope & deflection occurs at free end.

At $x=0m$ using eqn(1) valid upto 1st part

$$EI \left(\frac{dy}{dx} \right) = -20(0)^2 + 346.64$$

$$\therefore \text{Max. slope } \theta_B = \frac{346.64}{20} = \underline{\underline{17.332 \text{ rad.}}} \quad (1\text{M})$$

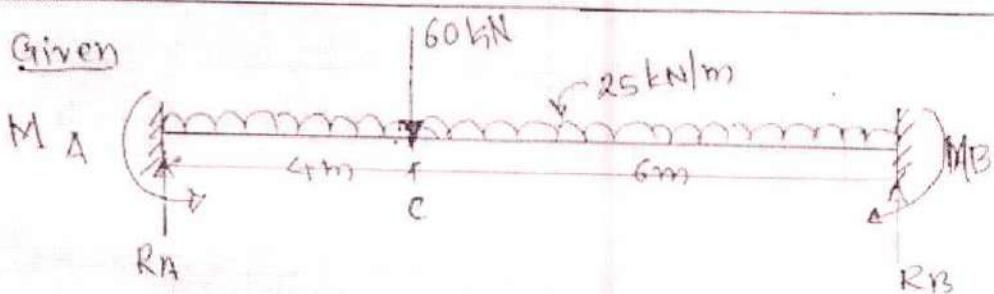
Using eqn(2) valid upto 1st part at $x=0m$,

$$EI(y) = -6.67(0)^3 + (346.64 \times 0) + (-946.56) \quad (1\text{M})$$

$$\therefore y_B = \frac{-946.56}{20}$$

$$\text{Max. deflection } y_B = \underline{\underline{-47.328 \text{ m}}} \quad (1\text{M})$$

Q.6 b)

GivenSol'n

i) From first principle, fixed end moment (Hanging)

is at bend,

$$MA = -\frac{25(10)^2}{12} - \frac{60(4)(6)}{(10)^2} = \underline{\underline{-294.73 \text{ kN.m}}} \quad (1\text{M})$$

$$MB = -\frac{25(10)^2}{12} - \frac{60(4)^2(6)}{(10)^2} = \underline{\underline{-265.93 \text{ kN.m}}} \quad (1\text{M})$$

ii) Calculation of Reactions using conditions of Eqn.,

$$\begin{aligned} \sum M_A = 0 = & -294.73 + 265.93 + (60 \times 4) \\ & + (25 \times 10 \times \frac{10}{2}) - (RB \times 10) \end{aligned}$$

$$\therefore R_B = 146.12 \text{ kN}$$

& $\sum F_y = 0 = R_A + R_B - 60 - (25 \times 10) = 0$

$$\therefore R_A = 163.88 \text{ kN}$$

(1m)

(1m)

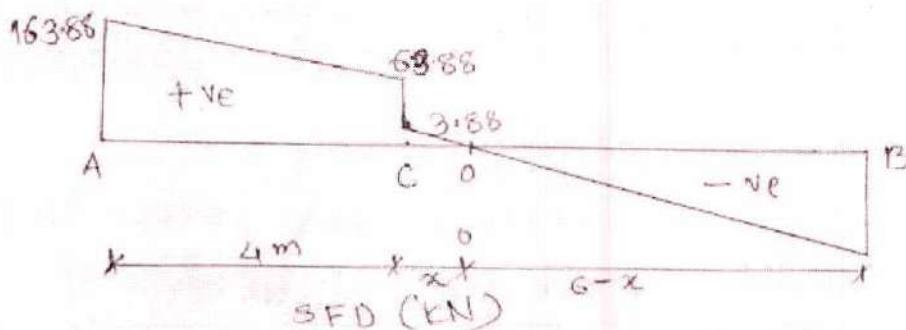
3) SF calculations:

$$SF \text{ at A} = +163.88 \text{ kN}$$

$$SF \text{ at C without } 60 \text{ kN} = 163.88 - (25 \times 4) = 63.88 \text{ kN}$$

$$SF \text{ at C with } 60 \text{ kN} = 63.88 - 60 = 03.88 \text{ kN}$$

$$SF \text{ at B} = -146.12 \text{ kN}$$



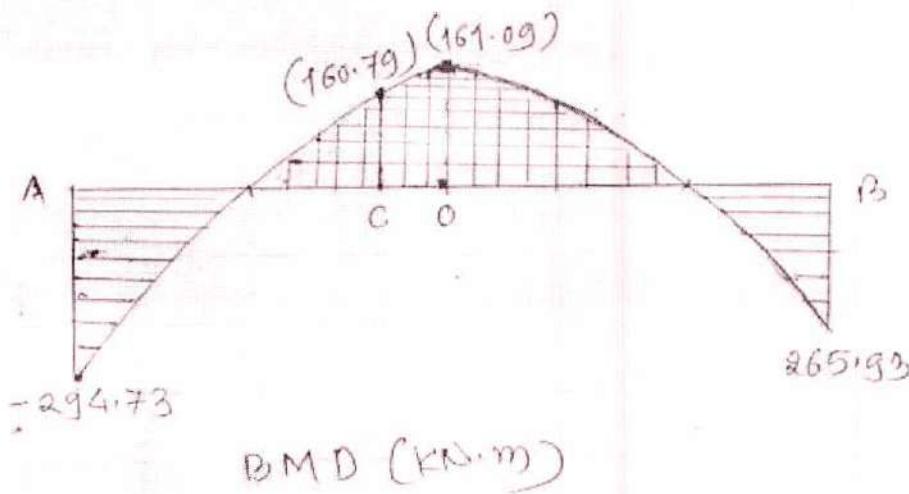
(1m)

$$\text{Position of pl 'O' from 'C'} \quad \frac{x}{3.88} = \frac{(6-x)}{146.12} \quad \therefore x = 0.155 \text{ m}$$

4) BM calculations

$$\text{Net BM at C} = -294.73 + (163.88 \times 4) - (25 \times 4 \times \frac{4}{2}) = 160.79 \text{ kNm}$$

$$\text{Net BM at 'O'} = -294.73 + (163.88 \times 4.155) - (25 \times \frac{(6+1.55)^2}{2}) \\ = 161.09 \text{ kNm}$$



(1m)

Q.No.	Question and Model Answers	Marks
Q.6C)	<p><u>Given</u></p> <p>(A) $\begin{array}{l} \text{P. } 2\text{m } \downarrow \\ \text{P. } 3\text{m } \uparrow \end{array}$ (B) $\begin{array}{l} \uparrow \text{ P. } 2\text{m } 3\text{m} \\ \downarrow \end{array}$ (C) $\begin{array}{l} \uparrow \text{ P. } 4\text{m} \\ \downarrow \end{array}$ (D) $\begin{array}{l} \uparrow \text{ P. } 4\text{m} \\ \downarrow \end{array}$</p> <p>Soln \rightarrow Calculation for sagging moment (+ve)</p> <p>for span AB = $\frac{120 \times 2 \times 4}{6} = 160 \text{ kN}\cdot\text{m}$ (1/2 M)</p> <p>for span BC = $\frac{80 \times 2 \times 3}{5} = 96 \text{ kN}\cdot\text{m}$ (1/2 M)</p> <p>for span CD = $\frac{50 \times 4^2}{8} = 100 \text{ kN}\cdot\text{m}$ (1/2 M)</p> <p></p> <p>$a_1 = \frac{1}{2} \times 6 \times 160$ $a_2 = \frac{1}{2} \times 5 \times 96$ $a_3 = \frac{2}{3} \times 4 \times 100$ (1/2 M)</p> <p>$a_1 = 480$ $a_2 = 240$ $a_3 = 266.67$ (1/2 M)</p> <p>Sagging moment area diagram.</p> <p>2) Using Three moment Theorem for pair of span.</p> <p>[PAIR A-B-C]</p> <p>$(M_A \times 6) + 2M_B(6+5) + (M_C \times 5) = -6 \left[\left(\frac{480 \times 2.67}{6} \right) + \left(\frac{240 \times 2.67}{5} \right) \right]$ (1/2 M)</p> <p>As Support A is SLS $\therefore M_A = 0$</p> <p>$22M_B + 5M_C = -2050.56 \dots \dots \textcircled{1}$ (1/2 M)</p> <p>[PAIR B-C-D]</p> <p>$(M_B \times 5) + 2M_C(5+4) + (M_D \times 4) = -6 \left[\left(\frac{240 \times 2.33}{5} \right) + \left(\frac{266.67 \times 2}{4} \right) \right]$ (1/2 M)</p>	

As support 'D' is S/S hence $M_D = 0$,

$$\therefore 5MB + 18MG = -1471.05 \quad \text{Eqn ②} \quad (kN)$$

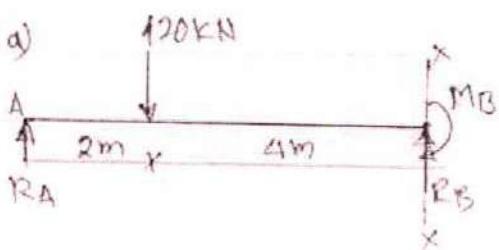
Solving eqn ① & ②

$$MB = -79.67 \text{ kNm} \quad \text{-ve sign indicated} \quad (kN)$$

$$\& MC = -59.58 \text{ kNm} \quad \text{Hogging BM.} \quad (kN)$$

b) Using Method of parts for finding support reactions.

a)



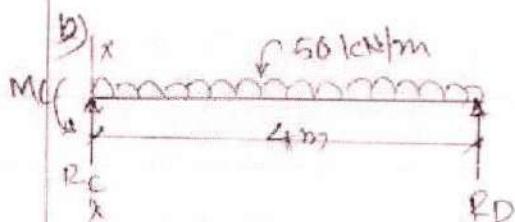
BM at section XX.

$$(RA \times 6) - (120 \times 4) = -79.67$$

$$RA = 66.72 \text{ kN}$$

(kN)

b)



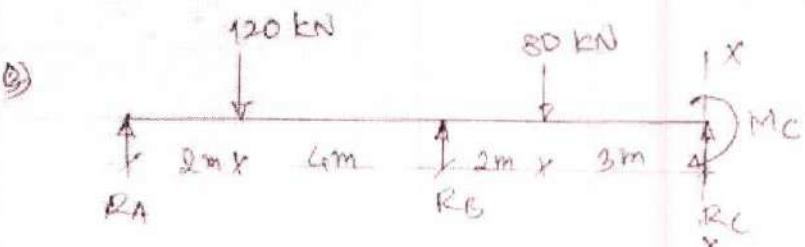
BM at section XX

$$(RD \times 4) - \frac{50 \times 4^2}{2} = -59.58$$

$$RD = 85.11 \text{ kN}$$

(kN)

c)



$$\text{BM at section XX} - (RA \times 11) + (RB \times 5) - (120 \times 9) - (80 \times 3) \\ = -59.58$$

\therefore

$$RB = 105.3 \text{ kN}$$

(kN)

$$d) \sum F_y = 0 = RA + RB + RC + RD - 120 - 80 - (50 \times 4)$$

$$\therefore RD = 142.87 \text{ kN}$$

(kN)